AIP 2025

Applied
Inverse
Problems

FGV EMAp, Rio de Janeiro
Brazil

Organized and hosted by

FGV | SCHOOL OF APPLIED MATHEMATICS

12º APPLIED INVERSE PROBLEMS CONFERENCE
JULY 28 - AUGUST 01, 2025

Speakers:

- **Tatiana Bubba** (University of Ferrara, Italy)
- **Jun Lai** (Zhejiang University, China)
- **Ru-Yu Lai** (University of Minnesota-Twin Cities, USA)
- **Mikyoung Lim** (Korea Advanced Institute of Science and Technology (KAIST), Republic of Korea)
- **Jennifer L. Mueller** (Colorado State University, USA)
- **Helcio R.B. Orlande** (Federal University of Rio de Janeiro, Brazil)
- **Axel Osses** (Center for Mathematical Modeling, Chile)
- **Eva Sincich** (University of Trieste, Italy)
- **Plamen Stefanov** (Purdue University, USA)

Local Committee:

- **Vinicius Albani** (UFSC, Brazil)
- **Liliane Barichello** (UFRGS, Brazil)
- **Adriano DeCezaro** (FURG, Brazil)
- **Alexandre Kawano** (USP, Brazil)
- **Antonio Leitão** (UFSC, Brazil)
- **Majela Machado** (UFBA, Brazil)
- **Fabio Margotti** (UFSC, Brazil)
- **Axel Osses** (Univ. of Chile, Chile)
- **Yuri Saporito (Chair)** (FGV EMAp, Brazil)

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Scientific Committee:

- **Simon Arridge** (Univ. College London, UK)
- **Uri Ascher** (UBC, Canada)
- **Elena Beretta** (NYU Abu Dhabi, UAE)
- **Liliana Borcea** (Michigan Univ., USA)
- **Fioralba Cakoni** (Rutgers Univ., USA)
- **Elisa Francini** (Università degli Studi di Firenze, Italy)
- **Thorsten Hohage** (Goettingen Univ., Germany)
- **Alfredo Iusem** (FGV EMAp, Brazil)
- **Barbara Kaltenbacher** (Klagenfurt Univ., AU)
- **Katya Krupchyk** (UCI, USA)
- **Antonio Leitão (Chair)** (UFSC, Brazil)
- **Shuai Lu** (Fudan Univ., China)
- **Carola-Bibiane Schönlieb** (Cambridge Univ., UK)
- **Cristiana Sebu** (Malta Univ., Malta)
- **Samuli Siltanen** (Helsinki Univ., Finland)
- **Tanja Tarvainen** (Univ. of Eastern Finland, Finland)
- **Gunther Uhlmann** (Univ. of Washington, USA)
- **Jorge Zubelli** (Khalifa Univ., Abu Dhabi)

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Organizing Team

Scientific Committee

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- ◇ Barbara Kaltenbacher, Klagenfurt Univ., Austria
- ◇ Katya Krupchyk, UCI, USA
- ◇ Antonio Leitão (**Chair**), Federal Univ. of St.Catarina, Brazil
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- ◇ Jorge Zubelli, Khalifa University, UAE

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- ◇ Vinicius Albani, Federal Univ. of St.Catarina, Brazil
- ◇ Liliane Barichello, Federal Univ. of Rio Grande do Sul, Brazil
- ◇ Adriano De Cezaro, Federal Univ. of Rio Grande, Brazil
- ◇ Alexandre Kawano, State Univ. of Sao Paulo, Brazil
- ◇ Antonio Leitão, Federal Univ. of St.Catarina, Brazil
- ◇ Majela Machado, Federal Univ. of Bahia, Brazil
- ◇ Fabio Margotti, Federal Univ. of St.Catarina, Brazil
- ◇ Axel Osses, Univ. of Chile, Chile
- ◇ Yuri Saporito (**Chair**), Getulio Vargas Foundation, Brazil

Venue

Getulio Vargas Foundation (FGV)
School of Applied Mathematics (EMAp)
Praia de Botafogo 190, 5th Floor
22250-900 Rio de Janeiro, Brazil

Webpage

https://eventos.fgv.br/aip2025

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Program

Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Registration 8:00-9:00				IPIA Assembly 8:00-9:00
Opening 9:00-9:50	**Calderon Talk** 9:00-9:50	**Plenary Talk** 9:00-9:50	**Calderon Talk** 9:00-9:50	**Parallel Session 7** 9:00-11:00
Plenary Talk 9:50-10:40	**Plenary Talk** 9:50-10:40	**Poster Sessions** 9:50-11:10	**Plenary Talk** 9:50-10:40	
Coffee Break 30min	Coffee Break 30min	Extended Coffee Break	Coffee Break 30min	Coffee Break 20min
Plenary Talk 11:10-12:00	**Plenary Talk** 11:10-12:00	**Plenary Talk** 11:10-12:00	**Plenary Talk** 11:10-12:00	**Plenary Talk** 11:20-12:10
Lunch Break 90min	Lunch Break 90min		Lunch Break 90min	Lunch Break 80min
Parallel Session 1 13:30-15:30	**Parallel Session 3** 13:30-15:30	Social Events 15:00-18:00	**Parallel Session 5** 13:30-15:30	**Parallel Session 8** 13:30-15:30
Coffee Break 30min	Coffee Break 30min	(free afternoon)	Coffee Break 30min	Coffee Break 30min
Parallel Session 2 16:00-18:00	**Parallel Session 4** 16:00-18:00		**Parallel Session 6** 16:00-18:00	**Parallel Session 9** 16:00-18:00
Welcome Icebreaker 18:00-20:00		Conference Dinner 19:00-21:00		

Plenary Talks

Plenary talks and Calderón talks will take place at the Cultural Center and will be streamed to the conference room on the 12th floor of FGV's main building.

Day (Time)	Speaker	Title
Monday (9:50-10:40)	Tatiana Bubba Univ. of Ferrara, Italy	On tomographic imaging with limited data
Monday (11:10-12:00)	Ru-Yu Lai Univ. of Minnesota-Twin Cities, USA	Inverse problems for kinetic equations
Tuesday (9:50-10:40)	Jun Lai Zhejiang Univ., China	Fast algorithms for forward and inverse multiple scattering problems
Tuesday (11:10-12:00)	Helcio R.B. Orlande Federal Univ. of Rio de Janeiro, Brazil	Inverse problems in magnetic resonance thermometry
Wednesday (9:00-9:50)	Plamen Stefanov Purdue Univ., USA	Seeing nonlinearities
Wednesday (11:10-12:00)	Eva Sincich Univ. of Trieste, Italy	Lipschitz stability for the determination of coefficients and elastic inclusions
Thursday (9:50-10:40)	Mikyoung Lim Korea Advanced Inst. of Sci. and Technology	Applications of geometric function theory to inverse problems
Thursday (11:10-12:00)	Axel Osses Univ. of Chile	Some inverse problems in reconstructing cardiac valve geometry and heart fiber orientation
Friday (11:20-12:10)	Jennifer Mueller Colorado State Univ., USA	Electrical impedance tomography in 2025

Parallel Sessions – Contributed Minisymposia

All rooms are located on the 3rd, 4th, and 5th floors of FGV's main building. Talks in minisymposia are 25 minutes long, followed by 5 minutes for questions.

MS-01 Celebrating Diversity in Inverse Problems
(Melody Alsaker, Gonzaga University & Siiri Rautio, University of Helsinki)

MS-02 Young researchers in geometric inverse problems and PDEs
(Antti Kykkänen, Rice University & Hjørdis Schlüter, University of Jyväskylä)

MS-03 Data Assimilation for Inverse Problems
(Neil Chada, City University of Hong Kong & Daniel Sanz Alonso, University of Chicago)

MS-04 Integral geometry, rigidity and geometric inverse problems
(Francois Monard, UC Santa Cruz & Plamen Stefanov, Purdue)

MS-05 Inverse Problems for Linear PDEs
(Boya Liu, North Dakota State Univ. & Teemu Saksala, North Carolina State Univ. & Lili Yan, Univ. of Minnesota)

MS-06 Imaging using coupled physics
(Teemu Sahlström, Univ. of Eastern Finland & Niko Hänninen, Univ. of Eastern Finland & Tanja Tarvainen, Univ. of Eastern Finland)

MS-07 Inverse Problems for Partial Differential Equations: Advances and Applications
(Kirill Golubnichiy, Texas Tech Univ.)

MS-08 New advances in data-driven numerical algorithms for inverse problems
(Davide Bianchi, Sun Yat-sen Univ.)

MS-09 Inverse Problems in Radiative Transport
(Simon Arridge, Univ. College London & Liliane Barichello, Federal Univ. of Rio Grande)

MS-10 Recent advances in computational inversion
(Tatiana Bubba, Univ. of Ferrara & Samuli Siltanen, Univ. of Helsinki)

MS-11 XAIP: explainable algorithms for inverse problems
(Giovanni Covi, Univ. of Helsinki & Matti Lassas, Univ. of Helsinki & Siiri Rautio, Univ. of Helsinki)

MS-12 Level Set Method and Neural Networks for solving Inverse Problems
(Cong Shi, Univ. of Vienna & Otmar Scherzer, Univ. of Vienna & Thi Lan Nhi Vu, Univ. of Vienna)

MS-13 Numerical linear algebra meets nonsmooth optimization in computed tomography
(Patricio Guerrero, KU Leuven & Mirjeta Pasha, Virginia Tech)

MS-14 Low dimensional representations in complex imaging problems
(Elena Loli Piccolomini, Univ. of Bologna & Serena Morigi, Univ. of Bologna)

MS-15 Breaking free from ground truth: beyond supervised learning in inverse problems
(Tatiana Bubba, Univ. of Ferrara & Luca Calatroni, Univ. of Genoa & Luca Ratti, Univ. of Bologna & Matteo Santacesaria, Univ. of Genoa)

MS-16 Inverse Problems and Control Theory with Applications in Biomedicine
(Axel Osses, Universidad de Chile & Benjamin Palacios, Pontificia Universidad Católica de Chile)

MS-17 Fast Numerical Methods for Forward and Inverse Scattering Problems
(Carlos Borges, Univ. of Central Florida)

MS-18 Variational regularization theory and applications
(Thorsten Hohage, Uni. Göttingen & Frank Werner, Uni. Würzburg)

MS-19 Convex Optimization Methods for Inverse Problems
(Dirk Lorenz, Uni. Bremen & Maicon Marques Alves, Federal Univ. of St. Catarina)

MS-20 Inverse Problems in Systems of Partial Differential Equations and Applications
(Alexandre Kawano, Universidade de São Paulo & Abdelmalek Zine, École Centrale de Lyon)

MS-21 Inverse Problems: PDEs and Integral Geometry
(Suman Sahoo, Indian Institute of Technology & Venky Krishnan, Tata Institute of Fundamental Research)

MS-22 Inverse problems on Riemannian and Lorentzian manifolds
(Matti Lassas, Univ. of Helsinki & Jinpeng Lu, Univ. of Helsinki & Teemu Saksala, North Carolina State Univ.)

MS-23 Imaging with Nonlinear Waves
(Barbara Kaltenbacher, Univ. of Klagenfurt & Vanja Nikolic, Radboud Univ.)

MS-24 Learned Optimisation Methods for Inverse Problems in Imaging
(Alexander Denker, Univ. College London & Zeljko Kereta, Univ. College London & Hok Shing Wong, Univ. of Bath)

MS-25 Recent trends and advances in Imaging: Models, Methods, and Applications
(Andrea Sebastiani, Univ. of Modena & Giorgia Franchini, Univ. of Modena & Silvia Tozza, Univ. of Bologna)

MS-26 Direct and Inverse Problems Arising in Quantitative Finance
(Vinicius Albani, Federal Univ. of St. Catarina & Yuri Saporito, EMAp/FGV)

MS-27 Levenberg-Marquardt methods in inverse problems: theory and applications
(Douglas Gonçalves, Federal Univ. of St. Catarina & Fermín Bazán, Federal Univ. of St. Catarina)

MS-28 Inverse Problems in Nano-Structures: Advances in Characterization, Modeling, and Applications
(Alexandre Kawano, Universidade de São Paulo & Antonino Morassi, Univ. of Udine)

MS-29 Inverse Problems for Evolution Equations
(Soumen Senapati, Karlsruhe Institute of Technology & Arpan Mukherjee, Shenzhen MSU-BIT Univ. & Manmohan Vashisth, Indian Institute of Technology)

MS-30 Probabilistic Learning Methods for Inverse Problems
(Edoardo Calvello, Caltech & Ricardo Baptista, Caltech & Nikola Kovachki, NVIDIA)

MS-31 Inverse problems for multiscale heterogeneous media
(Shari Moskow, Drexel Univ. & Fioralba Cakoni, Rutgers Univ.)

MS-32 On novel solution methods for inverse problems
(Joel Rabelo, Federal Univ. of Piaui & Antonio Leitão, Federal Univ. of St. Catarina)

MS-33 Analysis of Inverse Problems for Partial Differential Equations
(Andrea Aspri, Universit‘a degli Studi di Milano Statale & Eva Sincich, Università degli Studi di Trieste)

MS-34 Self-Supervised Techniques in Image Reconstruction
(Jürgen Frikel, Univ. Innsbruck & Markus Haltmeier, Univ. Innsbruck)

MS-35 Inverse Problems and Mathematical Models for the Analysis of Astrophysical data
(Paolo Massa, Univ. of Applied Sciences and Arts Northwest Switzerland & Anna Maria Massone, Università di Genova)

MS-36 Inverse Problems in Healthcare, Energy, and Industrial Applications
(Aster Santana, Mip Wise & Fábio Ramos, Federal Univ. of Rio de Janeiro)

MS-37 Inverse Problems and Machine Learning for Digital Health and Proximity Care
(Claudio Estatico, Univ. of Genoa)

MS-38 Optimal experimental design for inverse problems and related topics
(Christian Aarset, Univ. of Göttingen & Barbara Kaltenbacher, Univ. of Klagenfurt & Tram Nguyen, Max Planck Institute for Solar System Research)

Day (Time)	**Minisymposium**	**Part/Total**	**Room**
Monday (13:30-15:30)	MS-27 Levenberg-Marquardt Methods in Inverse Problems: Theory and Applications	1 / 2	306
	MS-03 Data Assimilation for Inverse Problems	1 / 2	307
	MS-09 Inverse Problems in Radiative Transport	1 / 2	308
	MS-19 Convex Optimization Methods for Inverse Problems	1 / 2	317
	MS-15 Breaking Free from Ground Truth: Beyond Supervised Learning in Inv. Probl.	1 / 2	318
	MS-21 Inverse Problems: PDEs and Integral Geometry	1 / 2	408
	MS-23 Imaging with Nonlinear Waves	1 / 2	409
	MS-22 Inverse problems on Riemannian and Lorentzian Manifolds	1 / 3	417
	MS-01 Celebrating Diversity in Inverse Problems	1 / 3	418
	MS-02 Young Researchers in Geometric Inverse Problems and PDEs	1 / 2	537
Monday (16:00-18:00)	MS-20 Inverse Problems in Systems of PDEs and Applications	1 / 1	306
	MS-03 Data Assimilation for Inverse Problems	2 / 2	307
	MS-09 Inverse Problems in Radiative Transport	2 / 2	308
	MS-19 Convex Optimization Methods for Inverse Problems	2 / 2	317
	MS-15 Breaking Free from Ground Truth: Beyond Supervised Learning in Inv. Probl.	2 / 2	318
	MS-21 Inverse Problems: PDEs and Integral Geometry	2 / 2	408
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	MS-02 Young Researchers in Geometric Inverse Problems and PDEs	2 / 2	537

Day (Time)	**Minisymposium**	**Part/Total**	**Room**
Tuesday (13:30-15:30)	MS-12 Level Set Method and Neural Networks for Solving Inverse Problems	1 / 2	306
	MS-06 Imaging Using Coupled Physics	1 / 2	307
	MS-17 Fast Numerical Methods for Forward and Inverse Scattering Problems	1 / 1	308
	MS-25 Recent Trends and Advances in Imaging: Models, Methods and Applications	1 / 3	317
	MS-24 Learned Optimisation Methods for Inverse Problems in Imaging	1 / 2	318
	MS-36 Inverse Problems in Healthcare, Energy, and Industrial Applications	1 / 2	408
	MS-35 Inv. Probl. and Mathematical Models for the Analysis of Astrophysical Data	1 / 2	409
	MS-22 Inverse problems on Riemannian and Lorentzian Manifolds	3 / 3	417
	MS-01 Celebrating Diversity in Inverse Problems	3 / 3	418
	MS-08 New Advances in Data-Driven Numerical Algorithms for Inverse Problems	1 / 1	537
Tuesday (16:00-18:00)	MS-12 Level Set Method and Neural Networks for Solving Inverse Problems	2 / 2	306
	MS-06 Imaging Using Coupled Physics	2 / 2	307
	MS-26 Direct and Inverse Problems Arising in Quantitative Finance	1 / 1	308
	MS-25 Recent Trends and Advances in Imaging: Models, Methods and Applications	2 / 3	317
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	MS-36 Inverse Problems in Healthcare, Energy, and Industrial Applications	2 / 2	408
	MS-35 Inv. Probl. and Mathematical Models for the Analysis of Astrophysical Data	2 / 2	409
	MS-10 Recent Advances in Computational Inversion	1 / 2	417
	MS-28 Inverse Problems in Nano-Structures: Advances in Characterization, Modeling and Applications	1 / 1	418
	MS-04 Integral Geometry, Rigidity and Geometric Inverse Problems	1 / 3	537

Day (Time)	Minisymposium	Part/Total	Room
Thursday (13:30-15:30)	MS-18 Variational Regularization Theory and Applications	1 / 2	306
	MS-38 Optimal Experimental Design for Inverse Problems and Related Topics	1 / 2	307
	MS-37 Inv. Probl. and Machine Learning for Digital Health and Proximity Care	1 / 1	308
	MS-25 Recent Trends and Advances in Imaging: Models, Methods and Applications	3 / 3	317
	MS-30 Probabilistic Learning Methods for Inverse Problems	1 / 1	318
	MS-32 On Novel Solution Methods for Inverse Problems	1 / 2	408
	MS-29 Inverse Problems for Evolution Equations	1 / 3	409
	MS-10 Recent Advances in Computational Inversion	2 / 2	417
	MS-31 Inverse Problems for Multiscale Heterogeneous Media	1 / 2	418
	MS-04 Integral Geometry, Rigidity and Geometric Inverse Problems	2 / 3	537
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	MS-38 Optimal Experimental Design for Inverse Problems and Related Topics	2 / 2	307
	MS-34 Self-Supervised Techniques in Image Reconstruction	1 / 1	317
	MS-32 On Novel Solution Methods for Inverse Problems	2 / 2	408
	MS-29 Inverse Problems for Evolution Equations	2 / 3	409
	MS-31 Inverse Problems for Multiscale Heterogeneous Media	2 / 2	418
	MS-04 Integral Geometry, Rigidity and Geometric Inverse Problems	3 / 3	537

Day (Time)	Minisymposium	Part/Total	Room
Friday (9:00-11:00)	MS-27 Levenberg-Marquardt Methods in Inverse Problems: Theory and Applications	2 / 2	306
	MS-13 Numerical Linear Algebra Meets Nonsmooth Optimization in Computed Tomography	1 / 2	307
	MS-33 Analysis of Inverse Problems for PDEs	1 / 2	308
	MS-07 Inverse Problems for PDEs: Advances and Applications	1 / 3	408
	MS-16 Inverse Problems and Control Theory with Applications in Biomedicine	1 / 2	409
	MS-11 XAIP: Explainable Algorithms for Inverse Problems	1 / 3	418
	MS-05 Inverse Problems for Linear PDEs	1 / 3	537
Friday (13:30-15:30)	MS-14 Low Dimensional Representations in Complex Imaging Problems	1 / 1	306
	MS-13 Numerical Linear Algebra Meets Nonsmooth Optimization in Computed Tomography	2 / 2	307
	MS-07 Inverse Problems for PDEs: Advances and Applications	2 / 3	408
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Parallel Sessions – Invited Minisymposia

All talks will take place at the Cultural Center. The talks in invited minisymposia are 25 minutes long, followed by 5 minutes for questions.

MS-39 Fortieth Anniversary of the Journal Inverse Problems (1985 - 2025)

Inverse Problems (iopscience.iop.org/journal/0266-5611) is a peer-reviewed, broad-based interdisciplinary journal for pure and applied mathematicians and physicists produced by IOP Publishing. It combines theoretical, experimental and mathematical papers on inverse problems with numerical and practical approaches to their solution. The journal has a specialized relevance to workers in geophysics, optics, radar, acoustics, communication theory, signal processing and medical imaging.

The editor-in-chief is Otmar Scherzer at University of Vienna, Austria. It is indexed in Applied Mechanics Reviews, INSPEC Information Services, ISI (Science Citation Index, SciSearch, ISI Alerting Services, COMPUMATH Citation Index, Current Contents/Physical, Chemical and Earth Sciences), Mathematical Reviews, Current Mathematical Publications, MathSciNet, Engineering Index/Ei Compendex, Zentralblatt MATH, and VINITI Abstracts Journal.

Day (Time)	Room
Thursday (13:30-15:30)	Cultural Center
— Fioralba Cakoni, Rutgers University, USA — Simon Arridge, University College London, UK	
Thursday (16:00-18:00)	Cultural Center
— Elena Beretta, NYU, Abu Dhabi — Otmar Scherzer, University of Vienna, Austria — Masahiro Yamamoto, University of Tokyo, Japan	

MS-40 Women in Inverse Problems Workshop (WiIP) at AIP

The objective of this event is to bring together women in the broad and vibrant field of Inverse Problems. Both established as well as early career researchers will come together to discuss their recent research achievements via a luncheon, a two-part minisymposium and poster session.

This meeting will facilitate professional networking and create mentoring opportunities for women researchers. The ultimate goal is to help broaden female participation in research careers in particular in the field of Inverse Problems, as well as to create new research collaborations.

Day (Time)	Activity	Room
Tuesday (12:00-13:30)	Women in IP Luncheon	T.B.A.
Tuesday (13:30-15:30)	Paralel Session	Cultural Center
Tuesday (16:00-18:00)	Paralel Session	Cultural Center
Wednesday (9:50-11:10)	Poster session WiIP	Foyer (Cultural Center)

Poster Sessions

The poster sessions will be held in the foyer of the Cultural Center on Wednesday, from 9:50 to 11:10 AM.

Contributed posters

Poster presenters: Rose Anne Alas, Lara Baalbaki, Arrianne Crystal Velasco, Abdelhamid Badran, Jonna Kangasniemi, Miika Suhonen, Felipe Riffel, Antonio Andrade, Juan Urueña, Jürgen Frikel, Marco Pauleti, Felix Schneppe, Marcos Obando, Seiji Hansen, Jon Kay, Zhengyi Qi.

WiIP posters

Poster presenters: Nasrin Nikbakht, Margaret Duff, Alessandra Serianni.

Detailed Conference Program

Monday, July 28, 2025

Plenary Talks

1 On tomographic imaging with limited data
Tatiana Bubba

2 Inverse problems for kinetic equations
Ru-Yu Lai

Parallel Session – 13:30 to 15:30

MS-27 Levenberg-Marquardt Methods in Inverse Problems: Theory and Applications – Room: 306

3 Levenberg-Marquardt method with singular scaling for zero residue problems: intuition and results
Everton Boos, Douglas Gonçalves and Fermin Bazan

4 Reconstruction of Transient Anisotropic Diffusion Tensor in Diffusion Models
Vanda Luchesi, Aline Lunkes and Everton Boos

5 Inversion of magnetic anomalies from dikes incorporating data error estimates
Saulo Oliveira, Rodoilton Stevanato, Alessandra Bongiolo and Eduardo Salamuni

6 Local convergence of Levenberg-Marquardt with singular scaling for nonzero residual problems
Douglas Gonçalves, Everton Boos, Fermin Bazan and Rafaela Filippozzi

MS-03 Data Assimilation for Inverse Problems – Room: 307

7 Uncertainty quantification for electrical impedance tomography using quasi-Monte Carlo methods
Laura Bazahica, Vesa Kaarnioja and Lassi Roininen

8 Nesterov Acceleration for Ensemble Kalman Inversion and Variants
Eviatar Bach, Sydney Vernon and Oliver Dunbar

9 Long-time accuracy of ensemble Kalman filters for chaotic and machine-learned dynamical system
Daniel Sanz-Alonso and Nathan Waniorek

MS-09 Inverse Problems in Radiative Transport – Room: 308

10 Inverse Analysis to Detect Skin Cancer Tissue from Transient Reflectance Signal of a Short-Pulse Laser Beam
Helcio Orlande, Cristiano H. da Silva Junior and Pedro J. Coelho

11 Inverse problem for a transport equation pencil beam approximation on slightly variable speed media
Matias Courdurier, Simon Arridge and Benjamin Palacios

12 On the Application of the ADO Method to the Coefficient Recovery Problem of the RTE
Fernando Groff, Liliane Barichello and Simon Arridge

13 On a regularization strategy for simultaneous reconstruction of optical coefficients from QPAT measurements in the radiative transfer regime
Adriano De Cezaro, Fabiana Travessini De Cezaro and Julian Sejje Suarez

MS-19 Convex Optimization Methods for Inverse Problems – Room: 317

14 Higher order error estimates for regularization of inverse problems under non-additive noise
Diana-Elena Mirciu and Elena Resmerita

15 Inexact splitting methods with relative errors
Dirk Lorenz, Maicon Marques Alves and Emanuele Naldi

16 Lyapunov analysis for FISTA under strong convexity
Luis Briceño-Arias

MS-02 Young Researchers in Geometric Inverse Problems and PDEs – Room: 537

Parallel Sessions – 16:00 to 18:00

MS-20 Inverse Problems in Systems of PDEs and Applications – Room: 306

MS-03 Data Assimilation for Inverse Problems – Room: 307

MS-09 Inverse Problems in Radiative Transport – Room: 308

MS-19 Convex Optimization Methods for Inverse Problems – Room: 317

MS-15 Breaking Free from Ground Truth: Beyond Supervised Learning in Inverse Problems – Room: 318

MS-21 Inverse Problems: PDEs and Integral Geometry – Room: 408

MS-23 Imaging with Nonlinear Waves – Room: 409

MS-22 Inverse Problems on Riemannian and Lorentzian Manifolds – Room: 417

MS-01 Celebrating Diversity in Inverse Problems – Room: 418

109 Inverse Problem of Parameter Calibration from Diverse Observations for Complex Hydrocarbon Mixtures
Hamidreza Anbarlooei, Reza Arefidamghani and Fabio Ramos

MS-35 Inverse Problems and Mathematical Models for the Analysis of Astrophysical Data – Room: 409

110 Viscous-inertial waves on the surface of the Sun: modeling, forward and inverse problems
Tram Nguyen, Damien Fournier and Thorsten Hohage

111 Variably Scaled Kernels for the regularized solution of the parametric Fourier imaging problem
Emma Perracchione, Anna Volpara, Anna Maria Massone and Michele Piana

112 Feature Understanding and Sparsity Enhancement via 2-Layered kernel machines (2L-FUSE)
Emma Perracchione, Fabiana Camattari, Sabrina Guastavino and Francesco Marchetti

113 3D reconstruction of solar flare emission
Barbara Palumbo, Paolo Massa, Daniel Ryan, Yang Su, Krucker Samuel, Anna Maria Massone, Federico Benvenuto and Michele Piana

MS-22 Inverse Problems on Riemannian and Lorentzian Manifolds – Room: 417

114 T.B.A
Pedro Caro

115 T.B.A
Yang Zhang

116 T.B.A
Gunther Uhlmann

MS-01 Celebrating Diversity in Inverse Problems – Room: 418

117 Variable Projection Methods for Large-scale Inverse Problems
Malena Español and Gabriela Jeronimo

118 Parameter Recovery in Inverse Gravimetry and EIT from Limited Measurements
Aseel Titi

MS-08 New Advances in Data-Driven Numerical Algorithms for Inverse Problems – Room: 537

119 Improved impedance inversion by iterated graph Laplacian
Florian Bossmann, Davide Bianchi and Wenlong Wang

120 A Preconditioned Version of a Nested Primal-Dual Algorithm for Image Deblurring
Giuseppe Scarlato, Stefano Aleotti, Marco Donatelli and Rolf Krause

121 A Learned Inverse Problem formulation for medical image enhancement under Contrast Agent Reduction
Elena Morotti, Davide Evangelista and Davide Bianchi

122 Priorconditioned Sparsity-Promoting Methods for Deterministic and Bayesian Linear Inverse Problems
Mirjeta Pasha

Parallel Sessions – 16:00 to 18:00

MS-40 Women in Inverse Problems Workshop (WiIP) – Cultural Center

123 Vertex characterization via second-order topological derivatives
Bochra Mejri-Mergl , Peter Gangl and Otmar Scherzer

124 Unique continuation and stability estimates for inverse boundary value problems
Elisa Francini, Sergio Vessella and Jenn-Nan Wang

125 Concentration inequalities and signal recovery
María Ángeles García-Ferrero and Joaquim Ortega-Cerdà

MS-35 Inverse Problems and Mathematical Models for the Analysis of Astrophysical Data – Room: 409

MS-10 Recent Advances in Computational Inversion – Room: 417

MS-28 Inverse Problems in Nano-Structures: Advances in Characterization, Modeling and Appl. – Room: 418

MS-04 Integral Geometry, Rigidity and Geometric Inverse Problems – Room: 537

Wednesday, July 30, 2025

Plenary Talks

Poster Sessions – 9:50 to 11:10

Contributed Posters – Foyer (Cultural Center)

WiIP Posters – Foyer (Cultural Center)

Thursday, July 31, 2025

Calderón Talk

Plenary Talks

Parallel Sessions – 13:30 to 15:30

MS-39 Fortieth Anniversary of the Journal Inverse Problems – Cultural Center

MS-18 Variational Regularization Theory and Applications – Room: 306

MS-38 Optimal Experimental Design for Inverse Problems and Related Topics – Room: 307

MS-37 Inverse Problems and Machine Learning for Digital Health and Proximity Care – Room: 308

MS-25 Recent Trends and Advances in Imaging: Models, Methods and Applications – Room: 317

Friday, August 1, 2025

Plenary Talk

Parallel Sessions – 9:00 to 11:00

MS-27 Levenberg-Marquardt Methods in Inverse Problems: Theory and Applications – Room: 306

MS-13 Numerical Linear Algebra Meets Nonsmooth Optim. in Computed Tomography – Room: 307

MS-33 Analysis of Inverse Problems for PDEs – Room: 308

MS-07 Inverse Problems for PDEs: Advances and Applications – Room: 408

MS-16 Inverse Problems and Control Theory with Applications in Biomedicine – Room: 409

MS-11 XAIP: Explainable Algorithms for Inverse Problems – Room: 418

MS-05 Inverse Problems for Linear PDEs – Room: 537

Parallel Sessions – 13:30 to 15:30

MS-14 Low Dimensional Representations in Complex Imaging Problems – Room: 306

MS-13 Numerical Linear Algebra Meets Nonsmooth Optim. in Computed Tomography – Room: 307

MS-07 Inverse Problems for PDEs: Advances and Applications – Room: 408

MS-16 Inverse Problems and Control Theory with Applications in Biomedicine – Room: 409

MS-11 XAIP: Explainable Algorithms for Inverse Problems – Room: 418

MS-05 Inverse Problems for Linear PDEs – Room: 537

Parallel Sessions – 16:00 to 18:00

MS-33 Analysis of Inverse Problems for PDEs – Room: 308

On tomographic imaging with limited data

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Keywords: tomography; sparse regularization; data-driven inversion

Tomographic imaging is an essential non-destructive technique in medicine and industry for visualizing internal structures of objects. A key challenge lies in the inherent noise and limitations (scarcity or undersampling) of measurements, requiring accurate modeling and the integration of prior information for reliable reconstructions. Consequently, the challenge of limited data tomography has spurred considerable theoretical and numerical investigation in recent years, with significant attention towards, inter alia, variational regularization and data-driven approaches.

This talk will highlight applications of limited data tomography involving both static and dynamic targets. In these scenarios, the inherent ill-posedness is tackled by leveraging the synergistic combination of sparse regularization theory, applied harmonic analysis, microlocal analysis tools, and (self-)supervised learning.

This talk is based on a series of joint works with T. Heikkilä, D. Labate, M. Lassas, S. Mukherjee, L. Ratti, M. Santacesaria, A. Sebastiani and S. Siltanen.

Inverse problems for kinetic equations

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Keywords: Kinetic equations; Inverse problems

Kinetic theory provides an effective description of the dynamics of a large number of interacting particles. One of the representative kinetic models is the Boltzmann equation. In this talk, I will review fundamental results for inverse kinetic problems, and also discuss some recent developments in this direction.

Levenberg-Marquardt method with singular scaling for zero residue problems: intuition and results

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Keywords: Levenberg-Marquardt; Singular Scaling Matrix; Convergence analysis; Parameter identification.

Inspired by certain regularization techniques for linear inverse problems, in this work we investigate the convergence properties of the Levenberg-Marquardt method using singular scaling matrices [2]. Under a completeness condition, we show that the method is well-defined and establish its local quadratic convergence under an error bound assumption. We also prove that the search directions are gradient-related allowing us to show that limit points of the sequence generated by a line-search version of the method are stationary for the sum-of-squares function. The usefulness of the method is illustrated with some examples of parameter identification in heat conduction problems for which specific singular scaling matrices can be used to improve the quality of approximate solutions [1, 3].

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Reconstruction of Transient Anisotropic Diffusion Tensor in Diffusion Models

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Keywords: Chebyshev pseudospectral method; Positive definite tensor; Levenberg-Marquardt method; Partial differential equation.

We will describe a transient diffusion model with initial and boundary conditions with the main goal of reconstructing the associated anisotropic transient diffusion tensor. The diffusion problem lies within a finite domain $(0, t_f] \times \Omega$, $t_f > 0$, with $\Omega \subseteq \mathbb{R}^d$, $d = 2, 3$ and is governed mainly by the equation

$$\frac{\partial U}{\partial t}(t, \bar{x}) = \nabla \cdot [K(t, \bar{x}) \nabla U(t, \bar{x})] + g(t, \bar{x}), \tag{1}$$

with $\bar{x} \in \Omega$, $g(t, \bar{x})$ a time-dependent source and $U(t, \bar{x})$ the temperature. We are concerned with the inverse problem, where the diffusion $K(t, \bar{x})$, for $t \neq 0$, is regarded as unknown and has to be estimated from temperature data or by the pressure in a transport problem in porous media. Many studies to identify the diffusion (or conductivity) tensor have been developed and reported in the literature for the non-transient case [2, 1]. Our study focuses on determining the transient diffusion $K(t, \bar{x})$, which we will impose to be a positive definite matrix, characterizing important examples, for instance, the crystal conductivity (see [3, p. 41]) or fusion plasma diffusion (see [1]).

The reconstruction process involves solving the governing partial differential equation (PDE) using a semi-discrete approach, in which the Chebyshev pseudospectral method (CPM) plays a central role in the spatial discretization [2]. Additionally, to address the numerical inversion problem, the transient diffusion is reconstructed through a nonlinear least squares optimization problem by employing the Levenberg-Marquardt method (LMM), with the Morozov discrepancy principle as a stopping criterion. To demonstrate the efficacy of this approach, the method is specifically applied to the reconstruction of the diffusion tensor in some benchmark tests and in synthetic experimental examples. The numerical results indicate a good agreement between the exact and reconstructed transient diffusion.

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Inversion of magnetic anomalies from dikes incorporating data error estimates

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Keywords: Levenberg-Marquardt; geophysics; potential field methods; dipping dike model; coefficient of variation

The dipping dike model is a common approach in the inversion of magnetic profiles, especially when the geologic or aeromagnetic map indicates the presence of dikes in the study area. We employ a Metropolis-Hasting algorithm combined with the Levenberg-Marquardt (LM) method to invert magnetic profiles assuming a model of multiple dikes. Typically, the inversion of magnetic profiles provides estimates for the depth to the top, half-width, and the horizontal location of the top center of each dike, among other physical and geometrical parameters. For the LM step, we compare some approaches to estimate the data weighting matrix $\mathbf{W}_d$, namely the case $\mathbf{W}_d = \mathbf{I}$, where there is absence of information about the data error, the classical approach $\mathbf{W}_d = \operatorname{diag}(1/\sigma_1, \ldots, 1/\sigma_m)$, where σ_i is the standard deviation of the i-th datum, and $\mathbf{W}_d = \operatorname{diag}(1/CV_1, \ldots, 1/CV_m)$, where $CV_i = \sigma_i/\mu_i$ is the i-th coefficient of variation (whereas μ_i denotes the i-th mean value). The latter approach was motivated by the field experiment, which was carried out in a study area where the geology is predominantly characterized by the NW-SE dike swarm in the Ponta Grossa Arch axis (Southern Brazil, Paraná State). In addition to the field example, we consider examples with synthetic data contaminated with non-uniform noise.

Local convergence of Levenberg-Marquardt with singular scaling for nonzero residual problems

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Keywords: Levenberg–Marquardt; Semi-norm regularizer; convergence analysis; nonzero residual; nonlinear least-squares

In a recent article [1] a Levenberg-Marquardt method (LMM) with Singular Scaling was proposed, analyzed and successfully applied in parameter estimation problems in heat conduction, providing approximate solutions of better quality than those of the classic LMM. In [1], a local convergence analysis was presented for the zero residual case. Now, we study the local behavior of the method when applied to nonzero residual nonlinear least-squares problems [2]. We show that the local convergence depends on the control of the linearization error of the gradient of the least-squares function as well as a suitable choice of the regularization parameter. In specific settings, superlinear convergence can still be achieved but, in general, even linear convergence can be ensured only when a combined measure of nonlinearity and residual size is small enough. Some examples illustrate how the method locally behaves in nonlinear problems with nonzero residual.

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Uncertainty quantification for electrical impedance tomography using quasi- Monte Carlo methods

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Keywords: quasi-Monte Carlo; electrical impedance tomography; Bayesian inversion; uncertainty quantification

The theoretical development of quasi-Monte Carlo (QMC) methods for uncertainty quantification of partial differential equations (PDEs) is typically centered around simplified model problems such as elliptic PDEs subject to homogeneous zero Dirichlet boundary conditions. In this talk, a theoretical treatment of the application of randomly shifted rank-1 lattice rules to electrical impedance tomography (EIT) will be presented. EIT is an imaging modality, where the goal is to reconstruct the interior conductivity of an object based on electrode measurements of current and voltage taken at the boundary of the object. This is an inverse problem, which we tackle using the Bayesian statistical inversion paradigm. As the reconstruction, we consider QMC integration to approximate the so-called conditional mean (CM) estimate of the unknown conductivity given current and voltage measurements.

Nesterov Acceleration for Ensemble Kalman Inversion and Variants

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Keywords: Ensemble Kalman Inversion; Nesterov Acceleration; Derivative-Free Optimization

Ensemble Kalman inversion (EKI) is a derivative-free, particle-based optimization method for solving inverse problems. It can be shown that EKI approximates a gradient flow, which allows the application of methods for accelerating gradient descent. Here, we show that Nesterov acceleration is effective in speeding up the reduction of the EKI cost function on a variety of inverse problems. We also implement Nesterov acceleration for two EKI variants, unscented Kalman inversion and ensemble transform Kalman inversion. Our specific implementation takes the form of a particle-level nudge that is demonstrably simple to couple in a black-box fashion with any existing EKI variant algorithms, comes with no additional computational expense, and with no additional tuning hyperparameters. This work shows a pathway for future research to translate advances in gradient-based optimization into advances in gradient-free Kalman optimization.

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Long-time accuracy of ensemble Kalman filters for chaotic and machine-learned dynamical systems

Nathan Waniorek

Keywords: Data assimilation; Ensemble Kalman filter; long-time accuracy

Filtering is concerned with online estimation of the state of a dynamical system from partial and noisy observations. In applications where the state is high dimensional, ensemble Kalman filters are often the method of choice. This talk will discuss new results on long-time accuracy of ensemble Kalman filters. We introduce conditions on the dynamics and the observations under which the estimation error remains small in the long-time horizon. Our theory covers a wide class of partially-observed chaotic dynamical systems, which includes the Navier-Stokes equations and Lorenz models. In addition, we prove long-time accuracy of ensemble Kalman filters with surrogate dynamics, thus validating the use of machine-learned forecast models in ensemble data assimilation. Joint work with Nathan Waniorek.

Inverse Analysis to Detect Skin Cancer Tissue from Transient Reflectance Signal of a Short-Pulse Laser Beam

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Keywords: Transient radiative transfer equation, Discrete ordinates method, Skin cancer, Markov chain Monte Carlo Method, Bayesian statistics

The possibility of detecting a cancerous tissue below the skin surface, from transient reflectance signal resulting from a short laser pulse, was demonstrated by forward numerical simulations in [1]. Here, an inverse problem of parameter estimation was solved with simulated measurements, in order to estimate properties of tissue layers representing the skin as a one-dimensional medium. The focus of this work was the detection of a cancerous tissue in one of the layers that represented the skin. Transient measurements of the radiation intensity leaving the skin surface after a picosecond laser pulse were considered available for the solution of the inverse problem. Measurement errors were assumed as additive and uncorrelated Gaussian variables, with zero means and constant standard deviations. The optical properties of the skin layers were estimated by solving the inverse problem within the Bayesian framework of statistics, with the Metropolis-Hastings algorithm of the Markov chain Monte Carlo method. The sensitivity coefficients of the measured dependent variables with respect to the different model parameters were examined. Linear-dependent optical properties of the layers with healthy tissues were either assumed as deterministically known or modeled as truncated Gaussian variables with small variances. On the other hand, optical properties of a layer of known thickness, which could represent a healthy or a cancerous tissue, were modeled as uniform random variables, with bounds that enclosed properties of different types of cancer, as well as healthy tissues. The Markov chains were started at values that approximately corresponded to healthy tissues for all layers. Stochastic simulations with the Metropolis-Hastings algorithm correctly tended to equilibrium distributions around the reference values used to generate the simulated measurements for all cases considered. Therefore, the properties of a healthy or a cancerous tissue in the layer of interest could be appropriately recovered. It was shown that the present approach has potential to be used as an early-diagnostic tool of skin cancer.

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Inverse problem for a transport equation pencil beam approximation on slightly variable speed media

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Keywords: Radiative Transport; Ray transforms

In this talk we will discuss a paraxial approximation to the linear transport equation in a highly peaked forward scattering regime, in a medium with slightly variable speed. The approximation gives rise to a propagation model that is a variation of the Fermi pencil-beam equation with an extra term. This modified equation admits an explicit solution, and measurements along all the lines traversing an unknown object allows for an inverse problem to be posed and analyzed, with the goal of recovering the unknown attenuation, speed and scattering coefficients.

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On the Application of the ADO Method to the Coefficient Recovery Problem of the RTE

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Keywords: Time-Dependent Radiative Transfer Equation; Analytical Discrete Ordinates Method; Coefficient Recovery; Adjoint-Based Gradient Computation; Integral Transforms.

Recovering absorption and scattering coefficients from boundary measurements is a fundamental problem in applications such as optical tomography and infrared remote sensing. When estimates are obtained through cost function minimization, selecting an appropriate forward operator becomes crucial. The radiative transfer equation (RTE) is recognized as one of the most accurate models for describing photon propagation in participating media. However, its mathematical complexity poses significant challenges for practical implementation, especially in inverse problems, where iterative schemes require repeated evaluations of the forward model to compute the cost function and its gradient.

In this context, one of the most widely used methods for solving the RTE is the discrete ordinates method, in which the angular variable is discretized to reduce the problem to a system of differential equations in the spatial variables. Although a classical approach, the discrete ordinates method continues to be refined and adapted to different contexts. Notably, the Analytical Discrete Ordinates (ADO) method, a spectral variant, provides analytical solutions in the spatial domain, reducing the associated eigenvalue problem to half the number of directions. These features, combined with rescaling strategies for exponential evaluations, have enabled accurate, stable, and efficient solutions in various scenarios, including media with variable refractive indices and multidimensional problems, where the method is combined with nodal techniques.

This work explores the application of the ADO method as a forward solver in inverse problems for recovering absorption and scattering coefficients. We derive analytical solutions for both the RTE and its associated adjoint equation. The time-dependent problem is reformulated using integral transforms, specifically Laplace or Fourier, assuming a one-dimensional plane-parallel multilayered medium with arbitrary anisotropic scattering and reflective boundary conditions. These solutions facilitate the evaluation of a cost function defined by the least squares error between measured and estimated partial fluxes across multiple integral transform parameters and yield a concise gradient expression with components expressed as Frobenius inner products. Preliminary numerical tests using synthetically generated noisy data and the L-BFGS-B optimization method demonstrate the approach's sensitivity and suggest that, when combined with regularization techniques, the formulation can be extended to more complex configurations.

On a regularization strategy for simultaneous reconstruction of optical coefficients from QPAT measurements in the radiative transfer regime

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Keywords: Joint identification; Absorption, Scattering, QPAT, radiative transfer

This presentation covers a regularization approach [1] aimed at jointly reconstructing piecewise constant absorption and scattering coefficients. These reconstructions are derived from measurements of solutions of the radiative transfer equation pertinent to Photoacoustic tomography imaging (QPAT) [2, 3]. Theoretical outcomes are illustrated with numerical examples utilizing synthetic data. Further details on these results are available in [4].

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Higher order error estimates for regularization of inverse problems under non-additive noise

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Keywords: Ill-posed problems, Variational regularization, Bregman distance

This talk focuses on higher order error estimates in terms of Bregman distances for inverse problems distorted by non-additive noise. The results are obtained by means of a novel source condition, inspired by the dual problem. Specifically, we focus on variational regularization having the Kullback-Leibler divergence as data-fidelity, and a convex penalty term. In this framework, we provide an interpretation of the new source condition, and present error estimates also when a variational formulation of the source condition is employed. We show that this approach can be extended to variational regularization that incorporates more general convex data fidelities.

Inexact splitting methods with relative errors

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Keywords: Convex optimization; splitting methods; preconditioned proximal point method

In this talk we apply the recently introduced framework of degenerate preconditioned proximal point algorithms to the hybrid proximal extragradient (HPE) method for maximal monotone inclusions. The latter is a method that allows inexact proximal (or resolvent) steps where the error is controlled by a relative-error criterion. Recently the HPE framework has been extended to the DouglasRachford method by Eckstein and Yao. In this paper we further extend the applicability of the HPE framework to splitting methods. To this end we use the framework of degenerate preconditioners that allows to write a large class of splitting methods as preconditioned proximal point algorithms. In this way, we modify many splitting methods such that one or more of the resolvents can be computed inexactly with an error that is controlled by an adaptive criterion. Further, we illustrate the algorithmic framework in the case of Chambolle-Pock's primal dual hybrid gradient method and the Davis-Yin's forward Douglas-Rachford method. In both cases, the inexact computation of the resolvent shows clear advantages in computing time and accuracy.

Lyapunov analysis for FISTA under strong convexity

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Keywords: Inertial algorithms; linear convergence; splitting algorithms; strong convexity

In this paper, we conduct a theoretical and numerical study of the *Fast Iterative Shrinkage-Thresholding Algorithm* (FISTA) [1] under strong convexity assumptions. We propose an autonomous Lyapunov function that reflects the strong convexity of the objective function, whether it arises from the smooth or non-smooth component. This Lyapunov function decreases monotonically at a linear rate along the iterations of the algorithm for a fixed inertial parameter. Our analysis demonstrates that the best theoretical convergence guarantees for FISTA in this context are obtained when the full strong convexity is treated as if it belongs to the smooth part of the objective. Within this framework, we compare the performance of forward-backward splitting (FBS) and several FISTA variants, and find that this strategy leads FISTA to outperform all other configurations, including FBS. Moreover, we identify parameter regimes in which FBS yields better performance than FISTA when the strong convexity of the non-smooth part is not leveraged appropriately.

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A strongly convergent inertial proximal algorithm with applications to variational inequalities

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Keywords: Monotone inclusions; Inertial Algorithms; Strong Convergence; Proximal-Point; Variational Inequalities.

We propose an inertial variant of the strongly convergent inexact proximal-point (PP) method of Solodov and Svaiter (2000) for monotone inclusions. We prove strong convergence of our main algorithm under less restrictive assumptions on the inertial parameters when compared to previous analysis of inertial PP-type algorithms, which makes our method of interest even in finite-dimensional settings. We also performed an iteration-complexity analysis and applied our main algorithm to variational inequalities for monotone operators, obtaining strongly convergent (inertial) variants of Korpolevich's extragradient, forward-backward and Tseng's modified forward-backward methods.

Iterative Importance Fine-Tuning of Diffusion Models

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Keywords: Diffusion Models; Self-Supervised Learning; Importance Sampling

Diffusion models are an important tool for generative modelling, serving as effective priors in applications such as imaging and protein design. A key challenge in applying diffusion models to downstream tasks is efficiently sampling from resulting posterior distributions, which can be addressed using the h-transform.

This talk presents a self-supervised algorithm for fine-tuning diffusion models by estimating the h-transform, enabling amortised conditional sampling. Our method iteratively refines the h-transform using a synthetic dataset resampled with path-based importance weights [1]. The resampling step follows prior on rejection-resampling steps for sampling from unnormalized probability density functions [2]. We demonstrate the effectiveness of this framework on class-conditional sampling and reward fine-tuning for text-to-image diffusion models.

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Deep Unrolling Graph-Network for Nonlinear Multi-Frequency Electrical Impedence Tomography

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Keywords: Electrical Impedance Tomography; variational networks; Proximal Regularized Gauss Newton; Graph Neural Networks

Multi-frequency Electrical Impedance Tomography (mfEIT) represents a promising biomedical imaging modality that enables the estimation of tissue conductivities across a range of frequencies [1]. Addressing this challenge, we present a novel variational network, a model-based learning paradigm that strategically combines the advantages and interpretability of classical iterative reconstruction with the power of deep learning. This approach integrates graph neural networks (GNNs) within the iterative Proximal Regularized Gauss Newton (PRGN) framework. By unrolling the PRGN algorithm, where each iteration corresponds to a network layer, we leverage the physical insights of nonlinear model fitting alongside the GNN's capacity to capture inter-frequency correlations. Notably, the GNN architecture preserves the irregular triangular mesh structure inherently used in the solution of the nonlinear forward model, enabling accurate reconstruction of overlapping tissue fraction concentrations.

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Stochastic Optimisation in Imaging Inverse Problems with Examples in CT, PET and Motion Compensated MR

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Keywords: Tomography; Magnetic Resonance Imaging; Motion Compensation; Stochastic Optimisation; Open-source Software

Despite substantial advancements in iterative regularisation algorithms for inverse problems, these methods are often criticised for their slow convergence and high memory demands. Traditional first-order solvers and their primal-dual variants operate deterministically, requiring the entire dataset to be processed in every iteration. This approach can be computationally intensive, particularly as scanners produce ever-larger high-resolution and multi-modal datasets. This challenge has motivated the development of algorithms that offer faster convergence rates and reduced per-iteration computational costs and memory requirements, many of which utilise stochastic optimisation techniques. Unlike traditional deterministic methods, stochastic approaches randomly select and process a subset of the acquired data. For suitable inverse problems, this can reduce the per-iteration computational costs, while still ensuring progress towards the solution.

This talk combines two projects. The first project (work done with: Letizia Protopapa, Johannes Mayer, Jeanette Schulz-Menger, Kris Thielemans, Christoph Kolbitsch and Edoardo Pasca) focuses on motion-corrected image reconstruction (MCIR) for fast and efficient cardiac magnetic resonance imaging (MRI) acquisition with predictable scan times. Data obtained in different phases of respiratory and cardiac motion can be used, and the scan duration is unaffected by changes in heart rate or irregular breathing patterns. Achieving high-quality reconstructions from MCIR data typically requires iterative optimisation algorithms with regularisation. Reconstruction times increase with the number of motion states, which is particularly relevant in cardiac MRI, where both cardiac and respiratory motion corrections are necessary to minimise motion artefacts [1]. We present a stochastic optimisation approach using the Stochastic Primal Dual Hybrid Gradient (SPDHG) [2] algorithm, which demonstrate faster convergence in fewer epochs compared to deterministic algorithm comparisons.

The second project (work done with: Evangelos Papoutsellis, Edoardo Pasca, Jakob Jørgensen, Sam Porter and Kris Thielemans) extends the open-source Python Core Optimisation Library [3, 4] with a unique and user-friendly API designed for efficient implementation and testing of stochastic optimisation techniques. This framework serves as an invaluable tool for researchers and practitioners in the field of imaging inverse problems, enabling easy prototyping and experimentation with various optimisation models. We demonstrate this with both CT and PET datasets, the latter using the Synergistic Image Reconstruction Software (SIRF) [5].

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Learning Monotone Operators: Application to Inverse Imaging Problems

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Keywords: Monotone operator, Optimization, Inverse Problem, Deep learning, Plud-and-Play

In this presentation, we discuss an approach for learning monotone neural networks through a newly defined penalization loss, that has been introduced in [1]. The trained networks can then be used within plug-and-play (PnP) frameworks, for solving classes of variational problems, specifically monotone inclusion problems, commonly encountered in image processing tasks. By using monotone operator theory, we can ensure the convergence of the resulting PnP iterations, and characterize the associated limit point as a solution to a monotone inclusion problem. In particular, monotone neural networks can be used within the Forward-Backward-Forward (FBF) algorithm, offering a solution even when the Lipschitz constant of the neural network is unknown.

We will illustrate the proposed approach on solving non-linear inverse problems. To achieve this, we initially formulate the problem as a variational inclusion problem. Subsequently, we train a monotone neural network to approximate an operator that may not inherently be monotone. Leveraging the FBF algorithm, we then show simulation examples where the non-linear inverse problem is successfully solved.

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Inverse problem for a time-dependent convection-diffusion equation in admissible geometries

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Keywords: Convection-diffusion equation; Partial boundary data; Admissible manifold

There has been several works on inverse problems related to the steady-state convection-diffusion equation in both Euclidean and Riemannian geometry settings. In this talk, we discuss the corresponding time-dependent problem within the Riemannian geometric framework. Specifically, we consider the partial data inverse problem for a time-dependent convection-diffusion equation on an admissible manifold. We prove that the time-dependent convection and density terms can be uniquely recovered, modulo a natural gauge invariance.

Tensor tomography using V-line transforms with vertices restricted to a circle

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Keywords: V-line transform; Tensor tomography; Inversion algorithms; Integral moments

In this talk, we discuss the problem of recovering symmetric m-tensor fields supported in a unit disk $\mathbb{D}$ from a set of generalized V-line transforms, namely longitudinal, transverse, and mixed V-line transforms, and their integral moments. We work in a circular geometric setup, where the V-lines have vertices on a circle, and the axis of symmetry is orthogonal to the circle. We present two approaches to recover a symmetric m-tensor field from the combination of longitudinal, transverse, and mixed V-line transforms. If time permits, we will explain both methods. We also derive inversion algorithms to reconstruct a symmetric m-tensor field from its first $(m+1)$ integral moment longitudinal/transverse V-line transforms.

Microlocal analysis of non-linear operators arising in Compton CT

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Keywords: non-linear tomography; edge recovery; microlocal analysis

We present a novel microlocal analysis of a non-linear ray transform, $\mathcal{R}$, arising in Compton Scattering Tomography (CST). Due to attenuation effects in CST, the integral weights depend on the reconstruction target, f, which has singularities. Thus, standard linear Fourier Integral Operator (FIO) theory does not apply as the weights are non-smooth. The V-line (or broken ray) transform, V, can be used to model the attenuation of incoming and outgoing rays. Through novel analysis of V, we characterize the location and strength of the singularities of the ray transform weights. In conjunction, we provide new results which quantify the strength of the singularities of distributional products based on the Sobolev order of the individual components. By combining this new theory, our analysis of V, and classical linear FIO theory, we determine the Sobolev order of the singularities of $\mathcal{R}f$. The strongest (lowest Sobolev order) singularities of $\mathcal{R}f$ are shown to correspond to the wavefront set elements of the classical Radon transform, and we use this idea and known results on the Radon transform to prove injectivity results for $\mathcal{R}$. In addition, we present novel reconstruction methods based on our theory, and we validate our results using simulated image reconstructions.

Tensor tomography for a set of generalized V-line transforms in $\mathbb{R}^2$

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Keywords: V-line transforms; Tensor tomography; Mellin transform

In this talk, we discuss the inverse problem of reconstructing a symmetric m-tensor field in $\mathbb{R}^2$ from a a set of generalized V-line transforms, namely longitudinal, mixed, and transverse V-line transforms. Specifically, we focus on recovering a symmetric m-tensor field $\mathbf{f}$ supported within a disk $\mathbb{D}_R$ of radius R centered at the origin. This recovery process is accomplished through a combination of the aforementioned generalized V-line transforms, using two different techniques for different sets of data.

Determination of ultrasonic properties using complex periodic solutions to the Westervelt equation

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Keywords: Nonlinear ultrasound, harmonic imaging

This work focuses on identifying the unknown physical properties governing ultrasound waves as described by the Westervelt equation. This equation accounts for both linear dissipation and a specific type of nonlinear behavior. Our goal is to prove the simultaneous determination of the wave speed, diffusivity and nonlinear coefficients. We demonstrate that by using complex-valued, time-periodic solutions, induced from the boundary at a sufficiently high frequency, these coefficients can be uniquely determined from knowledge of boundary measurements at the first and second harmonics.

Multicoefficient identification and multiharmonics with the Jordan-Moore-Gibson-Thompson equation

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Keywords: coefficient identification; uniqueness; Newton's method; acoustic nonlinearity parameter tomography; Jordan-Moore-Gibson-Thompson equation

The Jordan-Moore-Gibson-Thompson JMGT equation, a third order in time quasilinear PDE, is an advanced model in nonlinear acoustics that – as opposed to classical second order in time models such as the Kuznetsov and the Westervelt equation takes into account finite speed of propagation. In this talk we aim to highlight the fact that this allows to prove either local or linearized uniqueness of space dependent coefficients as relevant for imaging, such as nonlinearity parameter, sound speed, and attenuation parameter, from measurement of a single time trace of the acoustic pressure on the boundary. Simultaneous identification of pairs of these coefficients only requires two such measurements, provided the excitation is chosen appropriately. This is a setting relevant to several ultrasound based tomography methods. Our approach relies on the Inverse Function Theorem, which requires to prove that the forward operator is a differentiable isomorphism in appropriately chosen topologies. In case of a harmonic excitation, we can use a multiharmonic expansion to work in frequency domain and illustrate the multiplication of information by nonlinearity.

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Mathematical models for nonlinear ultrasound contrast imaging with microbubbles

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Keywords: ultrasound contrast imaging, Westervelts equation, Rayleigh–Plesset equation, wave-ODE coupling

Ultrasound contrast imaging is a specialized imaging technique that applies microbubble contrast agents to traditional medical sonography, providing real-time visualization of blood flow and vessels. Gas-filled microbubbles are injected into the body, where they undergo compression and rarefaction and interact nonlinearly with the ultrasound waves. Therefore, the propagation of sound through a bubbly liquid is a strongly nonlinear problem that can be modeled by a nonlinear acoustic wave equation for the propagation of the pressure waves coupled via the source terms to a nonlinear ordinary differential equation of RayleighPlesset type for the bubble dynamics. In this talk, we first look at the derivation of such coupled models based on constitutive laws and then show an existence result for a Westervelt–Rayleigh–Plesset system. Thirdly, we discuss numerical experiments on both single-bubble dynamics and the interaction of microbubbles with ultrasound waves. In this context, we also develop multiharmonic algorithms that allow for computationally more efficient simulations.

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Inverse scattering problems for non-linear wave equations

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Keywords: Inverse problems for non-linear waves; Scattering problem

We show that an inverse scattering problem for a semi-linear wave equation can be solved on a manifold having an asymptotically Minkowskian infinity, that is, scattering functionals determine the topology and the metric of the manifold up to a conformal transformation. The manifold on which the inverse problem is considered is allowed to have event horizons or several infinities of which at least one has to be of the asymptotically Minkowskian type. The results are applied also for cosmological models in space-times that have no particle horizons. To formulate the inverse problems we define a new type of data, non-linear scattering functionals, which are defined also in the cases when the classically defined scattering operator is not well defined. This makes it possible to solve the inverse problems also in the case when some of the incoming waves lead to a blow-up of the scattered solution. The talk is based on the paper [1].

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Lipschitz Stability of Travel Time Data

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Keywords: geometric inverse problems; length spaces, GromovHausdorff distance; isometric embeddings

In this talk we show that the reconstruction of certain type of length spaces from their travel time data on a closed subset is Lipschitz stable. The travel time data is the set of distance functions from the entire space, measured on the chosen closed subset. The case of a Riemannian manifold with boundary with the boundary as the measurement set appears is a classical geometric inverse problem arising from Gelfands inverse boundary spectral problem. Examples of spaces satisfying our assumptions include some non-simple Riemannian manifolds, Euclidean domains with non-trivial topology, and metric trees.

Inverse problems for time-dependent nonlinear transport equations

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Keywords: Nonlinear transport equation; inverse problems; time-dependent coefficient; geometric optics solutions; light ray transform

In this talk, we will discuss inverse problems of recovering the time-dependent coefficient in the nonlinear transport equation in both cases: two-dimensional Riemannian manifolds and Euclidean space $\mathbb{R}^n$, $n \geq 2$. Specifically, it is shown that its initial boundary value problem is wellposed for small initial and incoming data. Moreover, the time-dependent coefficient appearing in the nonlinear term can be uniquely determined from boundary measurements as well as initial and final data. To achieve this, the central techniques we utilize include the linearization technique and the construction of special geometrical optics solutions for the linear transport equation. This allows us to reduce the inverse coefficient problem to the inversion of certain weighted light ray transforms. Based on the developed methodology, the inverse source problem for the nonlinear transport equation in the scattering-free media is also studied.

The Distribution of Scattering Phase Shifts of Magnetic Schrödinger Operators on Asymptotically Hyperbolic Manifolds

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Keywords: Inverse spectral problems on manifolds

We will discuss trace formulæ which establish a quantum-classical correspondence between the high energy limit of the scattering phase shifts of Schrödinger operators with real valued electric and magnetic potentials and their geodesic X-ray transforms on nontrapping asymptotically hyperbolic manifolds– in this setting the potentials are lower order semiclassical perturbations. These extend the results of Bulger and Pushnitski [1, 2] for electric and magnetic Schrödinger operators in Euclidean space, and the speaker [4] for the case of hyperbolic space. As an application, in the special case of simply connected asymptotically hyperbolic manifolds with non-positive sectional curvature, we prove that the high energy limit of the scattering phase shifts uniquely determine certain types of radial electric potentials with additional symmetries. This extends a result of Levinson [3] for radial potentials in Euclidean space to a special class of potentials on Cartan–Hadamard asymptotically hyperbolic manifolds.

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On the interplay between the light ray and the magnetic X-ray transforms

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Keywords: Light ray transform; magnetic X-ray transform; stationary Lorentzian manifold; integral geometry

A stationary Lorentzian manifold is associated to a Riemannian manifold equipped with a magnetic vector field. We study the light ray transform acting on tensors on a stationary Lorentzian manifold. Our main result is injectivity of the light ray transform up to the natural obstruction as long as the associated magnetic vector field satisfies a finite degree property with respect to the vertical Fourier decomposition on the unit tangent bundle. This is based on an explicit relationship between the geodesic vector field of the Lorentzian manifold and the magnetic vector field on the Riemannian manifold.

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Extending Qualitative Methods to Biharmonic Scattering

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Keywords: Direct Sampling Method ; Biharmonic Waves ; Transmission Eigenvalues

In this talk, we will discuss extending the direct sampling method (DSM) to inverse shape problems for biharmonic scattering. This method is a computationally simple and analytically rigorous way to define an imaging function to recover the scatterer. Here, we will focus on the case of scattering by a clamped region by an incident plane wave. Using the far-field data, we will analyze the DSM and prove its a stable reconstruction method. The techniques for analyzing the DSM are similar to the standard Helmholtz equation with a few interesting surprises we will discuss how to tackle. Numerical examples are given to show the applicability of the imaging functionals. We will also discuss the associated transmission eigenvalue problem.ă

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Real-data EIT reconstruction using virtual X-rays and deep learning

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Keywords: electrical impedance tomography; deep learning

The mathematical model of electrical impedance tomography (EIT) is the inverse conductivity problem introduced by Calderón. The aim is to recover the conductivity σ from the knowledge of the Dirichlet-to-Neumann map Λ_σ. It is a nonlinear and highly ill-posed inverse problem.

We introduce a new reconstruction algorithm for EIT, which provides a curious connection between EIT and traditional X-ray tomography, based on the idea of *virtual* X-rays. Using the method, EIT data can be processed into a virtual parallel-beam sinogram, that can then be reconstructed using, for example, filtered backprojection, which is a standard reconstruction algorithm for X-ray tomography.

We divide the exponentially ill-posed and nonlinear inverse problem of EIT into separate steps. We start by mathematically calculating so-called virtual X-ray projection data from the measurement data. Then, we perform explicit algebraic operations and one-dimensional integration, ending up with a blurry and nonlinearly transformed Radon sinogram. We use a neural network to remove the higher-order scattering terms and perform deconvolution. Finally, we can compute a reconstruction of the conductivity using the inverse Radon transform. We demonstrate the method with experimental data.

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Toward Virtual X-ray CT Reconstructions from Human Data Measurements

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Keywords: Virtual Hybrid Parallel-Beam Tomography; Electrical Impedance Tomography; Computed Tomography

The newly-developed method of virtual hybrid parallel-beam tomography (VHPT) involves computation of a virtual sinogram, with geometry identical to the sinogram normally obtained from parallel-beam X-ray CT measurements, but without the use of X-rays. Instead, we use harmless electrical currents as measurement energy, and a mathematical transformation based in microlocal analysis is applied to achieve a straight-line geometry. Reconstructions of experimental tank data collected on circular domains have been presented previously, but to date the method has not been applied to human data, which involves multiple challenges. This talk presents recent advancements toward virtual X-ray CT scans from real-world human data, including the first-ever preliminary reconstructions from human thoracic data.

Combining electrical impedance tomography and machine learning for stroke classification

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Keywords: Electrical impedance tomography; complete electrode model; virtual hybrid edge detection; neural networks; stroke classification

Stroke is a leading cause of disability and death all around the world. There are two main types of stroke: ischemic (blood clot preventing blood flow to a part of the brain causing a low-conductivity region) and hemorrhagic (bleeding in the brain causing a high-conductivity region). Both types of strokes present very similar symptoms in visual clinical inspection, but treatments are very different. Therefore, having portable equipment capable of classifying strokes would be very useful.

Electrical impedance tomography (EIT) is a non-invasive imaging method for recovering information about the internal conductivity of a physical body from electric boundary measurements. EIT combined with machine learning has shown promise for the classification of strokes. However, most previous works have used raw EIT voltage data as network inputs. In this work, EIT measurements are processed in a non-linear manner for exctracting relevant geometric information about conductivity. Then, these features, called Virtual Hybrid Edge Detection (VHED) functions [1], are used as input of neural networks. Different virtual patients are created, and simulated noisy EIT electrode data is used for the classification of stroke. The performance of neural networks using raw data is compared to machine learning based on the interpretable features. The results show that VHED functions outperform raw data as inputs for machine learning, showing promise for the stroke classification task.

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Several Equivalent Source-to-Solution Inverse Problems on Complete Riemannian Manifolds

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Keywords: Spectral Theory

Let (N, g) be a complete Riemannian manifold (potentially not compact). If $\mathcal{L} : C^\infty(N) \to C^\infty(N)$ is a symmetric operator known as the Magnetic-Schrödinger operator, then we can consider the hyperbolic problem

$$\begin{cases} (\partial_t^2 + \mathcal{L})u^f(t, x) = f(t, x), & \text{for } (t, x) \text{ in } (0, \infty) \times N \\ u^f(0, x) = \partial_t u^f(0, x) = 0, & \text{for } x \text{ in } N. \end{cases}$$

If $\mathcal{S}, \mathcal{R} \subset N$ are open sets then we can define the hyperbolic local source-to-solution operator $\Lambda^{hyp}_{\mathcal{S},\mathcal{R}}$ to be the map for $f \in C_0^\infty((0, \infty) \times \mathcal{S})$ to the solution u^f restricted to $(0, \infty) \times \mathcal{R}$. That is

$$\Lambda^{hyp}_{\mathcal{S},\mathcal{R}}(f) = u^f|_{(0,\infty)\times\mathcal{R}}.$$

We can similarly define a parabolic local source-to-solution operator $\Lambda^{par}_{\mathcal{S},\mathcal{R}}$ and a family of elliptic operators $\Lambda^{ellip}_{\mathcal{S},\mathcal{R},z}$. We shall show that knowledge of any of the operators is equivalent to knowledge of any of the other operators.

Lorentzian Calderón problem on vector bundles near Minkowski geometry

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Keywords: Inverse problems; Calderón problem, Lorentzian geometry, Wave equation

We discuss a version of the Calderón problem for the connection Laplacian

$$P = \nabla^* \nabla + V(t,x),$$

acting on sections of a Hermitian vector bundle E over a fixed Lorentzian manifold (M,g). Under the assumption that (M,g) admits a smooth, proper, surjective temporal function and satisfies a curvature bound, we show that the connection ∇ and potential V are uniquely determined by the Dirichlet-to-Neumann map, up to the natural group of gauge transformations. In particular, the result is applicable to (M,g) that are small perturbations of Minkowski geometry. This work builds on earlier results in the scalar setting [1] [2].

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The Non-Abelian X-Ray Transform on Asymptotically Hyperbolic Spaces

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Keywords: Non-Abelian X-Ray Transform; Asymptotically Hyperbolic Spaces

The inverse problem of medical CT scans is to recover an image of the X-ray absorption coefficient ϕ inside a patient from absorption data collected after irradiating the patient with X-rays. We consider a generalization of this problem that turns the scalar X-ray absorption equation into a coupled linear system (where the coefficient ϕ is now a square matrix) which asks whether the coefficient ϕ is still recoverable. This finds application in a form of imaging called polarimetric neutron tomography. In addition, we consider the mentioned problem on a class of unbounded geometries called asymptotically hyperbolic spaces. We demonstrate that under certain regularity conditions the coefficient ϕ in this case is also recoverable. If the coefficient is of the form $\phi + A$ where A is a matrix that depends linearly on velocity (e.g. comes from a connection), then the problem isnt solvable but the pair (ϕ, A) can be recovered up to a gauge.

A Calderóns problem for harmonic maps

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Keywords: First keyword; Second keyword

I will present a version of Calderóns problem for harmonic maps between Riemannian manifolds. In more detail, given (M, g) and (N, h) compact Riemannian manifolds with boundary, let $u \in C^\infty(M, N)$ be an harmonic map with boundary value $f \in C^\infty(\partial M, N)$, that is, in local coordinates, u solves

$$\begin{cases} \Delta_g u^i + g^{\alpha\beta}\Gamma^i_{jk}\partial_\alpha u^j \partial_\beta u^k = 0, \quad i = 1, \dots, \dim N, \\ u|_{\partial M} = f. \end{cases}$$

From the normal derivatives of u at the boundary, we ask if to what extent we can recover information on the metrics g and h. By using the higher linearization method, we first show that the Dirichlet-to-Neumann map determines the metric on the domain up to a natural gauge in three cases: on surfaces, on analytic manifolds, and in conformally transversally anisotropic manifolds on a fixed conformal class with injective ray transform on the transversal manifold. Next, using higher linearizations we obtain integral identities that allows us to show that the metrics on the target have the same jets at one point. In particular, if the target is analytic, the metrics are equal. We also prove an energy rigidity result, in the sense that the Dirichlet energies of harmonic maps determines the Dirichlet-to-Neumann map.

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Timoshenko Beam, Uniqueness Resultas and Forces Identification

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Keywords: Timoshenko Beam; force Indentification

We investigate the beam equation derived from Timoshenko Beam Theory (TBT), which models the dynamics of an elastic beam by accounting for both shear deformation and rotational inertiafeatures not captured by the classical Euler-Bernoulli theory. The TBT framework describes two primary quantities: the transverse displacement field u and the cross-sectional rotation φ. For a beam of length $L > 0$, with positive material constants G, E, ρ and positive geometric constants A, I, κ, the governing system is given by:

$$\begin{cases} \rho A \dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial}{\partial \xi}\left[AG\kappa\left(\dfrac{\partial u}{\partial \xi} - \varphi\right)\right] = g(t) f_1, & \text{in } (0,+\infty)\times(0,L), \\ \rho I \dfrac{\partial^2 \varphi}{\partial t^2} - \dfrac{\partial}{\partial \xi}\left(EI\dfrac{\partial \varphi}{\partial \xi}\right) - \kappa AG\left(\dfrac{\partial u}{\partial \xi} - \varphi\right) = g(t) f_2, & \text{in } (0,+\infty)\times(0,L), \end{cases}$$

where the beam is subjected to a time-dependent load of the form $g(t)(f_1(x), f_2(x))$. We show that, for any known function $g \in \mathcal{C}^1$ and given an observation subdomain $\omega \subset (0, L)$, there exists a time $T > 0$ such that the knowledge of $u(t, x)$ on $(0, T] \times \omega$ uniquely determines the spatial loading terms $(f_1, f_2) \in H^{-1}(\omega) \times H^{-1}(\omega)$.

Some numerical results will be presented. Numerical results demonstrate that the Timoshenko model outperforms the Bernoulli model. It more accurately captures the complexities of real behavior. However, it also exhibits greater sensitivity to measurement errors compared to the Bernoulli model.

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Versatility of Carleman estimates: application to inverse problems for Maxwell's system

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Keywords: Maxwell Systems; Carleman estimates

In this presentation, through the use of various Carleman estimates, I will produce several results on the simultaneous reconstruction of coefficients associated with Maxwell systems, using a finite number of observations. Furthermore, the hypotheses on the coefficients and the observation domains, allow to address a wide range of concrete situations.
This talk is inspired by the following papers [1], [2], [3].

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A Fluid-Structure interaction inverse problem

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Keywords: Fluid-structure interaction, Uniqueness, Wave equation, Euler-Bernoulli equation, Source identification

We investigate an inverse source problem in a coupled fluid-structure interaction (FSI) framework, in which a compressible fluid occupies a rectangular domain bounded below by an elastic Euler–Bernoulli beam. The system is excited by two unknown spatial sources with known time dependence: one acting in the fluid, the other on the beam. We aim to determine both spatial components from partial measurements of the beam displacement over a finite time interval. The mathematical model consists of a coupled system of wave and beam equations with appropriate interface and boundary conditions. Under minimal geometric assumptions and suitable regularity hypotheses, we establish a uniqueness result for the recovery of the source pair from restricted observations. Numerical simulations confirm the theoretical findings, highlighting the role of a minimal observation time and the sensitivity of the reconstruction to noise in the data. These results contribute to the mathematical understanding of inverse problems in vibroacoustics and provide a foundation for practical applications such as structural health monitoring and source localization.

Recent advances on inverse elasticity problems applied to material and medical imaging

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Keywords: Inverse parameter problems, linear elasticity, waveguides, medical imaging.

In this talk, I will present some recent advances in inverse problems involving the linear elastic system of equations in the context of non-destructive testing of materials and of medical elastography.

Elastic excitation responses of mechanical plates and waveguides are very sensitive to shape defects or material defects, in particular near the resonant frequencies. We tackle identification problems of recovering the shape defects or small internal inclusions from measurements of the wave field at the surface of the guide. We show that near the waveguide's resonant modes, there exists some "locally resonant" frequencies. At these frequencies there are three types of modes: prorogative, evanescent and quasi-resonant. It appears that, near the defect, the quasi-resonant mode satisfies an equation close to a"tunneling" Schrödinger equation. This particular mode can be represented explicitly using Airy functions. It is extremely sensitive to the defect and we show how it can be used for its accurate recovery [1, 2].

Medical elastography is a modern harmless medical imaging modality that aims at imaging the mechanical properties of living tissues to detect pathologies at early stage. Some displacement fields, generated by an external source, are measured inside the body using an auxiliary imaging method (such as ultrasound imaging, MRI, OCT,...). We then face the general inverse problem of recovering an elastic tensor field from some solutions of the linear elastic system of equations. Some recent improvements have been made in [3, 4] concerning the analysis of the Reverse Weak Formulation of this problem:

$$\text{Find} \quad \boldsymbol{C}(x) \quad \text{s.t.} \quad \int_\Omega (\boldsymbol{C} : \mathcal{E}(\boldsymbol{u})) : \mathcal{E}(\boldsymbol{v}) = \int_\Omega \boldsymbol{f} \cdot \boldsymbol{v}, \quad \forall \boldsymbol{v} \in H_0^1(\Omega, \mathbb{R}^d).$$

I will present theoretical stability results and an efficient approach to discretize this problem. This allows for accurate recovery of anisotropic tensor maps from quasi-static displacements. This approach has been applied with success to experimental data.

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Convergence Aspects of Hybrid Kernel SVGD

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Keywords: Approximate Inference; Particle-based Variational Inference; Gradient Flow

Stein variational gradient descent (SVGD) is a particle based approximate inference algorithm. Many variants of SVGD have been proposed in recent years, including the hybrid kernel variant (h-SVGD), which has demonstrated promising results on image classification with deep neural network ensembles. By framing h-SVGD as a kernelised Wasserstein gradient flow on a functional that is not the Kullback-Leibler divergence, we demonstrate that h-SVGD does not converge to the target distribution in the mean field limit. Despite this theoretical result, we provide intuition and experimental support for the ability of h-SVGD to improve variance estimation in high dimensions. Unlike other SVGD variants that also alleviate variance collapse, this is achieved at no additional computational cost and without further assumptions on the posterior.

Transformers for Data Assimilation

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Keywords: Attention; Transformers; Smoothing; Filtering.

Attention mechanisms and their use in transformers, the neural network architectures underlying the success of large language models, have been widely successful at modeling nonlocal correlations in data. Recent interest in applying the transformer architecture to scientific machine learning motivates the development of rigorous mathematical analysis for this methodology. In this talk we show how the mathematical properties of the attention mechanism enable the design of transformer architectures well suited for data assimilation tasks. Namely, we demonstrate how the formulation of attention as a map between spaces of functions allows for methodology performing state estimation in dynamical systems; similarly, a formulation of attention as a map between spaces of probability measures allows for methodology performing stochastic filtering. We show the promise of this novel approach to a variety of data assimilation contexts.

DIMP Approach to Computing the Radiation Source Distribution from the Responses of a Few Detectors

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Keywords: Radiation Source Energy & Space Distribution; Bayesian Inference

Generally, solving inverse problems raises difficult questions: Does a solution exist? Is it unique? How sensitive is it to small model/data perturbations? We address these by deploying Bayes Theorem expressing the input data as a distribution of values and computing the inverse problem's solution distribution. Our new approach, termed Data Integration with Modeled Predictions (DIMP), also yields confidence estimates in the resulting solution.

We use DIMP to estimate the energy and spatial distribution of a radioactive source from measured detector responses at a few locations within a given geometric/material configuration. Applications of DIMP include locating radioisotopes clandestinely hidden or during cleanup operations, and accounting for special nuclear material in fuel processing facilities, the so-called holdup problem. DIMP discretizes the physical domain into N computational cells and energy into G groups, which yields an NG vector of source intensities that produces a gamma flux at each detector, contributing to the response according to the detector response function. We focus on the uncollided photons response manifested as peaks in the measured spectrum. Now the inverse problem can be stated as: *Find the state in the state-space with maximum probability of observance given the measured detector responses.* The search for that state is informed by the discrepancy between the measured and modeled responses, with the latter computed as the scalar product of a state with the adjoint flux for the subject detector. To enhance computational efficiency, the adjoint gamma flux is computed as a preprocess for each deployed detector using the ray-tracing capability of the Denovo code and the results are catalogued in a database.

DIMP's initial implementation utilized a Quasi-Newton line search algorithm and was tested using puck gamma sources [1]. DIMP successfully located the sources to within 0.25 to 0.9 m on the room's 4×4 m^2 floor, but weakness of the sources limited its accuracy and caused the search to get stuck in local minima. To address these issues, we generated synthetic responses with MCNP models and enhanced DIMP with a basin-hopping global search algorithm to enable global optimization. While this improved the performance compared to the initial local-search version of DIMP for point sources it raised new issues [2]. A second measurement campaign was conducted at Oak Ridge National Laboratory using more powerful sources including disc and line HEU sources in bare and hidden configurations. DIMP located the shielded source near the true location, although it incorrectly predicted that some of the source was outside of the pipe.

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Radiative Transport of Entangled Photons

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Keywords: quantum optics, radiative transport

The study of classical light in random media has reached a certain degree of maturity. In contrast, the propagation of quantum states of light in random media is relatively unexplored. This talk will present a survey of recent works in this direction with applications to imaging.

On Randomised Reconstruction Strategies in Optical Tomography

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Keywords: Optical Tomography;Radiative Transport Equation; Monte Carlo; Stochastic Gradient Methods

Optical tomography is an imaging technique that reconstructs the internal structure of an object by analyzing how light is transmitted, scattered, and absorbed as it passes through. In highly scattering media, light propagation is often modeled using the diffusion approximation. However, when scattering is weak or in regions near sources and boundaries, the diffusion model becomes inaccurate. In such cases, the radiative transport equation (RTE) provides a more precise description of light propagation in turbid media. Due to the high computational cost of solving the RTE directly, Monte Carlo methods are widely used to simulate light transport. They offer flexibility and accuracy, and converge to the solution of the RTE in the limit of the number of simulated photons.

In this work, we consider a variational framework to estimate the solution. The goal is to minimise a data misfit functional regularised by prior information, and both the forward and inverse problems are addressed stochastically.

Reduced Order Models for Inverse Transport Problems

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Keywords: inverse transport; reduced-order models

One of the main challenges in inverse radiative transport problems is their high computational cost due to the high-dimensionality nature of the radiative transport equation. In this talk, I will discuss some computational strategies based on reduced-order models to accelerate inverse transport computations. While the methods are mainly motivated by physical intuitions, we will provide some justifications in simplified settings.

Algorithms with learned deviations

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Keywords: Convex and non-smooth optimization; Data-driven optimization

Deviations are a way to modify existing optimization algorithms. In contrast to, e.g., momentum terms parametrized by a single number, they can have the same dimensions as the optimization variables. Traditionally interpreted as unwanted errors, they can be used to tailor an algorithm to a specific class of problems, e.g., variationally regularized inverse problems with a given forward operator.

By choosing the deviations small enough, convergence rates can be guaranteed.

Our bounds for the norms of the deviations are based on known quantities, not summability. The performance of specialized algorithms for certain problem classes can be improved by training the deviations with deep learning. This talk is based on joint work with Jevgenija Rudzusika, Ozan Öktem, Jonas Adler, Hamed Sadeghi, Pontus Giselsson, and Oskar Bircks.

Relaxed and Inertial Nonlinear Forward-Backward with Momentum

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Keywords: Operator splitting; inertial methods

In this talk, we study inertial algorithms for numerically solving monotone inclusions involving the sum of a maximally monotone and a cocoercive operator. In particular, we analyze the convergence of inertial and relaxed versions of the nonlinear forward-backward with momentum (NFBM). Moreover, by applying NFBM to specific monotone inclusions, we derive inertial and relaxed versions of algorithms such as forward-backward, forward-half-reflected-backward (FHRB), Chambolle–Pock, Condat–Vũ, among others. We also present numerical experiments on image restoration, comparing the inertial and relaxed FHRB with its non-inertial and momentum version.

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Predictor-Corrector Strategy for Nonconvex Problems with Application to 1D Signal Decomposition

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Keywords: Nonconvex optimization; Predictor–corrector; Signal decomposition; Multi-parameter selection

Many real-world problems naturally lead to nonconvex optimization formulations. However, standard iterative solvers often struggle with these, being both slow and highly sensitive to initialization. We propose a predictor–corrector strategy that efficiently computes locally optimal solutions without requiring a good initial guess. The process begins with a convex minimization step, yielding a globally reliable preliminary candidate. This is followed by a parameter-free nonconvex refinement step. Surprisingly, even simple solvers—such as the Alternating Direction Method of Multipliers (ADMM)—perform remarkably well within this framework. We showcase the effectiveness of our approach on the challenging task of 1D signal decomposition, separating signals into semantically meaningful components: smooth, piecewise-constant, oscillatory, and unstructured (noise) elements.

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A Bregman proximal point analysis for a class of accelerated convex optimization algorithms

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Keywords: Bregman proximal mappings, Nesterov-type acceleration, adaptive algorithms

We propose a proximal point framework for accelerated convex optimization using Bregman proximal mappings as preconditioners. The latter generalize the common proximal map by replacing the squared-norm term in the associated minimization problem by the Bregman distance induced by a strictly convex, smooth and strongly coercive functional. We show that this not only allows for more flexibility in terms of preconditioners for proximal-point algorithms, but also enables a refined Bregman-based analysis of acceleration strategies, in particular of Nesterov type. We further demonstrate how the use of multiple Bregman-type preconditioners yields accelerated, in terms of worst-case rates for the error in the objective functional, as well as novel Bregman proximal point algorithms that are also amenable to adaptivity. The behavior of instances of these algorithms for the regularized solution of inverse problems is exemplarily shown and discussed via numerical experiments.

Robust accelerated PGET reconstruction for safe disposal of spent nuclear fuel

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Keywords: Nonlinear tomography; PGET; Iterative learning

Passive Gamma Emission Tomography (PGET)[1] is a nonlinear imaging method capable of accurately reconstructing the emission and attenuation profiles of radioactive spent nuclear fuel assemblies before their long-term underground disposal.

In this work we experiment with accelerating the existing iterative reconstruction algorithm using suitable machine learning methods [2]. Due to the tight safety concerns of the application, the aim is to retain as robust and explainable algorithm as possible which we do by restricting the machine learning updates using the update steps from the traditional algorithm. This slows the convergence but makes all tested methods very robust and well generizable to data outside the limited training samples.

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Unsupervised graphLaNet for imaging applications

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Keywords: Ill-posed problems; Graph Laplacian; Deep learning

Differential operators are popular choices for regularizing ill-posed problems in imaging. We investigate a variational method which embeds a graph Laplacian operator in the regularization term. The novelty of this method lies in constructing the graph Laplacian based on a preliminary approximation of the solution [1, 2, 3], which is obtained by an unsupervised neural network. As a result, the regularization term is both dependent on and adaptive to the observed data and noise. Under very mild assumptions, this method is regularizing and stable.

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Ambient Diffusion Omni: Training Good Models with Bad Data

Giannis Daras[1], Adrian Rodriguez-Munoz[2], Adam Klivans[6], Antonio Torralba[5], Costis Daskalakis[7], Jeffrey Ouyang-Zhang[3], and Daniel Jesus Diaz[4]

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Keywords: generative models, inverse problems, ambient diffusion, corrupted data, noisy data

We show that it is possible to use low-quality, corrupted, and out-of-distribution data to improve the quality of a diffusion model. Typically, diffusion models are trained on curated datasets that emerge from highly filtered data pools from the Web and other sources. We show that there is immense value in the lower quality data that is often discarded. We present Ambient Diffusion Omni, a simple, principled framework to train diffusion models that use arbitrarily corrupted and out-of-distribution data. We experimentally validate our framework in four distinct settings: i) we train pixel and latent-diffusion models with images synthetically corrupted by Gaussian blur, JPEG compression, and motion blur, ii) we improve the quality of state-of-the-art latent diffusion models and text-to-image models by utilizing better the low-quality parts of the training dataset, iii) we show that we can leverage out-of-distribution data to improve the performance of diffusion models in limited data settings, iv) we achieve state-of-the-art in protein structure generation using synthetic data from AlphaFold.

A Plug-and-Play block proximal heavy ball method for resource constrained scenarios

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Keywords: Plug-and-Play; Block Proximal Method; Resource Constrained

The interpretation Plug-and-Play (PnP) methods as the minimization of a certain objective function is guaranteed under suitable assumptions on the denoiser. In particular, Gradient Step (GS) denoisers [2] define the denoiser as the gradient of a potential function depending on the output of a convolutional neural network. This gradient is computed via backpropagation routines, which do not scale efficiently with image size in terms of memory occupation, often resulting in the overflow of the GPU memory. In this talk, we propose a novel memory-efficient optimization framework for PnP approaches, introducing a block proximal variant of an heavy ball method [1] that enables effective image reconstruction in resource-constrained settings. In particular, considering image patches as the blocks of the scheme, the memory occupation for the denoising is significantly reduced when performed on a single block. Under mild assumptions it is possible to derive convergence guarantees, characterizing the limit points. The experimental results confirm the efficacy of our method across multiple imaging problems including deblurring and super-resolution, showing the reduced cost for each iteration. The proposed framework allows to use PnP reconstruction techniques also in limited-memory scenarios, without sacrificing performance or convergence properties.

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Simultaneous Reconstruction of the Density, Bulk and Source in the Time-Domain Wave Equation

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Keywords: Time-Domain Wave Equation; Simultaneous reconstruction; Acoustic Resonators

In this talk, we will discuss an inverse problem in the time-domain wave equation which amounts to simultaneously reconstruct the medium properties (such as mass density and bulk modulus) and source function. For the data, along with traditional measurement i.e. the Dirichlet trace of wave-field on the boundary, we consider an auxiliary measurement in terms of the boundary observation of wave-field generated by the medium when it is injected by small-scaled contrast agents at different interior points. Under critical scaling of these contrast agents, we derive an asymptotic profile for the auxiliary wave-field which results in simultaneous reconstruction of material properties and source. Our analysis primarily uses the time-domain Lippmann-Schwinger equation and integral equation techniques.

This talk will be based on the work [1].

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A simple range characterization for spherical mean transform in even dimensions

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Keywords: Spherical mean transform; range characterization; Bessel functions; Nicholson-type identity; elliptic integrals

We discuss a new and simple range characterization for the spherical mean transform of functions supported in the unit ball in even dimensions. It complements the previous work of the same authors, where an analogous problem in odd dimensions was solved. The range description in even dimensions consists of symmetry relations, using a special kind of elliptic integrals involving the coefficients of the spherical harmonics expansion of the function in the range of the transform. We also introduce a pair of original identities involving normalized Bessel functions of the first and the second kind. The first result is an integral cross-product identity for Bessel functions of integer order, complementing a similar relation for Bessel functions of half-integer order obtained in the aforementioned work of the same authors. The second result is a new Nicholson-type identity. Both of these relations can be considered as important standalone results in the theory of special functions. Finally, we derive an interesting equality involving elliptic integrals, which may be of independent interest.

The matrix weighted real-analytic double fibration transforms

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Keywords: Radon transform; geodesic ray transform; nonabelian ray transform; analytic microlocal analysis

In this talk, we present a microlocal result showing that the real-analytic matrix-weighted double fibration transform determines the analytic wavefront set of a vector-valued function. As an application, we prove the injectivity of the matrix-weighted ray transform on two-dimensional, non-trapping, real-analytic Riemannian manifolds with strictly convex boundary. Furthermore, we show that a real-analytic Higgs field can be uniquely recovered from the nonabelian ray transform on real-analytic manifolds of any dimension, provided the manifold has a strictly convex boundary point.

The fixed angle inverse scattering problem

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Keywords: Fixed angle scattering; Hyperbolic inverse problem

Wave propagation in an inhomogeneous acoustic medium may be modeled, for example, by the wave operators $\Box + q(x)$, $\rho(x)\partial_t^2 - \Delta$ or $\partial_t^2 - \Delta_g$, for a function $q(x)$, a positive function $\rho(x)$ or a Riemannian metric $g(x)$, which are homogeneous outside a ball. The medium is probed by plane waves coming from a finite number (dimension dependent) of dierent directions, and the resultant time dependent waves are measured on the boundary of the ball. We describe our partial results about the recovery of q, ρ, g from these boundary measurements. These are long standing formally determined open problems.

A Hybrid Gaussian Beam Method for the Wave Equation

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Keywords: Wave Equation, Gaussian Beams, High Frequency Approximation

Gaussian Beams are high-frequency asymptotic solutions to the wave equation. When one considers a linear wave equation, a linear superposition of Gaussian Beams can be used to efficiently solve this. Motivated by the work of Qian and Ying [1], we present a hybrid multiscale Gaussian Beam method to solve the initial value problem for the linear wave equation, which uses `k-Wave` [2] to efficiently and accurately represent the low frequency components. We also present a related hybrid solver to solve the Dirichlet boundary value problem for the linear wave equation [3]. Finally, we consider applications of this method in nonlinear settings [4].

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Undetermined coefficient problems for reaction diffusion equations

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Keywords: Undetermined coefficients, nonlinear parabolic equations.

As opposed to linear parabolic equations with unknown coeffcients and terms, inverse problems for reaction-diffusion equations offer considerable additional complexities. This can go beyond just the inherent nonlinearities of the forward pde operator. For example, they can exhibit: "blow up" in finite time; or "quenching (whereby the solution rapidly approaches a constant value in finite time); or have travelling wave solutions in space-time.
In addition, the degree of coupling of the unknown terms and the nonlinearity can play a crucial role in the complexity of the inverse problem. We shall look at some examples of the above in this talk.

Partial data inverse problems for the nonlinear magnetic Schrödinger equation

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Keywords: Nonlinearity; Time-dependent magnetic Schrödinger equation

In this talk, we will discuss how to apply the knowledge of the Dirichlet-to-Neumann map, measured on an arbitrary part of the boundary, to uniquely determine the time-dependent linear coefficients, electric and magnetic potentials, and nonlinear coefficients.

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Lorentzian scattering rigidity, recent results

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Keywords: Lorentzian geometry; scattering and lens rigidity

I will present some recent results about the recovery of a Lorentzian metric, up to isometries and conformal multipliers, from the scattering relation on a cylinder-like timelike boundary. The first result concerns stationary metrics. The second one is about the recovery of the jet at the boundary, and local recovery of analytic metrics. The talk is based on the works below.

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Partial data inverse problems for the biharmonic operator with first order perturbation

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Keywords: Stability; biharmonic operator; partial data

We consider an inverse boundary value problem for the biharmonic operator with the first order perturbation in a bounded domain of dimension three or higher. Assuming that the first and the zeroth order perturbations are known in a neighborhood of the boundary, we establish log-type stability estimates for these perturbations from a partial Dirichlet-to-Neumann map. Specifically, measurements are taken only on an arbitrarily small open subsets of the boundary.

Anisotropic fractional Calderón problem with external data

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In this talk, we will discuss the fractional anisotropic Calderón problem with external data in Euclidean space, in dimensions two and higher, for smooth Riemannian metrics that agree with the Euclidean metric outside a compact set. We will show that the partial exterior Dirichlet–to–Neumann map for the fractional Laplace–Beltrami operator, known on arbitrary nonempty open subsets of the exterior of a domain in Euclidean space, determines the Riemannian metric up to a diffeomorphism that fixes the exterior. This is joint work with Ali Feizmohammadi, Tuhin Ghosh, Angkana Rüland, Johannes Sjöstrand, and Gunther Uhlmann.

Learned enclosure method for experimental EIT data

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Keywords: Electrical impedance tomography; Non-linear inverse problems, Neural networks

Electrical impedance tomography (EIT) is a non-invasive imaging method with diverse applications, including medical imaging and non-destructive testing. The inverse problem of reconstructing internal electrical conductivity from boundary measurements is nonlinear and highly ill-posed, making it difficult to solve accurately.

In this talk, we present a method for estimating the convex hull of inclusions from boundary measurements by combining the enclosure method proposed by Ikehata with neural networks. We demonstrate its performance using experimental data. Compared to the classical enclosure method with least squares fitting, the learned convex hull achieves superior performance on both simulated and experimental data.

How did I end up here and how are inverse problems related?

Emilia Blåsten[1]

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Keywords: anecdotes; open problems

I will describe challenges in my life, how mathematics provided solutions and how I ended up in the inverse problems community. I will present a few interesting anecdotes and memorable mathematical problems from along the way. I will present some of the most interesting bits of my mathematical results, including open problems and work that I have rarely talked about. Due to time constraints I will not have time to talk about each of them, but I will mention at least work on the 1D wave equation and general resolvent estimates and what's the core of idea in the proofs.

Recovering the metric fom relative distance comparison data

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Keywords: relative distance comparison, metric geometry, Riemannian geometry

Let (X, d) be a metric space. Suppose that we do not know the metric d, but we are given the following relation:

$$E = \{(x, y, z) \mid d(x, y) \leq \min \{d(x, z), d(y, z)\}\}.$$

The triplet (x, y, z) is in E if and only if x and y are at least as close to each other as they are to z. In laymann's terms, out of (x, y, z), z is the outlier. We show the metric d can be reconstructed up to a constant factor from the relation E in the case that X is a complete Riemannian manifold and d is its Riemannian distance metric. This talk is based on ongoing work.

Bayesian inversion of CT data to characterize transport in the mouse ear

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Keywords: Bayesian inverse problem, transport model, CT data, CUQIpy

Recent studies in rodents and humans show that gene therapy agents or tracers injected into the cerebrospinal fluid (CSF) reach the inner ear. The communication of fluid between the cochlear and the subarachnoid spaces of the brain has been controversial for decades. Observations of transport between the cochlea and the subarachnoid space contrast with the different composition of the fluids. The recent discovery of a membrane in the cochlear aqueduct raises further questions about the restrictions of transport between the compartments. This study aims to numerically quantify the diffusive and advective modes of transport of inert molecules from CSF to an intact cochlea. We use imaging data of the transport of a small tracer (Omnipaque, X kDa) through the cochlear aqueduct and scala tympani in five sedated mice (8-week-old males). To estimate the transport model parameters, we formulate and solve a Bayesian inverse problem in which we allow the diffusivity to vary in the presence of potential membranes. We also discuss how modeling choices affect the inference. We carry out the implementation using the software tool CUQIpy (Computational Uncertainty Quantification for Inverse Problems in Python).

Analytic Fourier Integral Operators and Inverse Problems

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Keywords: Analytic Fourier integral operator, analytic wavefront set, Bolker condition, acoustic wave equation

We motivate basic results of analytic microlocal analysis, in particular that it provides us with tools with which one can control the support of distributions using microlocal information. We exploit this observation to present a general ansatz for establishing uniqueness results for inverse problems that can be employed when the forward operator is in a general class of analytic Fourier integral operators. To demonstrate the efficacy of this approach, we discuss a problem from seismic inversion that can be tackled with this method.

Twistor spaces and inverse problems

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Keywords: X-ray tomography, transport equations, twistor spaces

If (M, g) is an oriented Riemannian surface, then the unit disk bundle $Z = \{(x, v) \in TM : |v| \leq 1\}$ carries a natural complex structure that degenerates at the unit circle bundle $SM \subset \partial Z$ in a way that encodes the geodesic flow. We refer to the complex surface Z as *transport twistor space*, in analogy with the more classical *projective twistor space* $Z_{\mathbb{P}} = Z/(x, v) \sim (x, -v)$, which has been used in the context of Zoll structures and projective structures.

Many inverse problems in 2-dimensions (e.g. tensor tomography, scattering rigidity, non-Abelian X-ray tomography) are commonly and very effectively phrased as questions about the transport equation on SM. As it turns out, one can often go one step further and interpret these as questions about the complex geometry of Z. This allows to organise and reinterpret many classical results in inverse problems, provides useful analogies (e.g. with Zoll structures or transparent connections) and gives rise to new intriguing questions about twistor spaces.

In the talk we will touch upon the first two aspects and then describe recent work pertaining to the last one. Using tools from inverse problems, we construct suitable *'blow-down maps'* $\beta \colon Z \to \mathbb{C}^2$ and classify *biholomorphisms* $Z \cong Z'$ between different transport twistor spaces.

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On the stability of a hyperbolic inverse problem

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Keywords: Hyperbolic inverse problems; Boundary Control method; unique continuation

The Boundary Control method is one of the main techniques in the theory of inverse problems. It allows to recover the metric or the potential of a wave equation in a Riemannian manifold from its Dirichlet to Neumann map (or variants) under very general geometric assumptions. In this talk we will address the issue of obtaining stability estimates for the recovery of a potential in some specific situations. As it turns out, this problem is related to the study of the blow-up of quantities coming from control theory and unique continuation. This is based on joints works with Lauri Oksanen.

T.B.A.

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Fast algorithms for forward and inverse multiple scattering problems

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Keywords: Multiple scattering; Inverse scattering; Boundary integral equations; Fast algorithms

Wave scattering and inverse scattering have important applications in geophysical exploration, medical imaging, nondestructive testing, and many others. How to achieve fast solutions to the wave equations is one of the widely concerned issues in computational mathematics. Integral equations provide an effective computational tool for solving wave scattering and inverse scattering problems, particularly for those involving multiple scattering effect. However, their numerical implementation faces challenges such as the discretization of singular integrals and the solution of dense linear systems. In this talk, we present an efficient computational approach for wave scattering by multiple particles, combining integral equations, scattering matrices, and the Fast Multipole Method (FMM) to overcome these challenges. We will also discuss the focusing theory and numerical methods for the inverse scattering problems involving multiple particles.

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Inverse Problems in Magnetic Resonance Thermometry

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Keywords: Temperature measurements; Thermal cancer treatment; Bayesian statistics; Thermometry image artefacts.

The temperature increase of tumor tissues is a cancer treatment currently available in clinical practice worldwide. Modern individualized cancer treatments require advanced planning, through mathematical modeling and computational simulations under different sources of uncertainties. Furthermore, temperature measurements during thermal cancer treatments can be non-intrusively taken by different methods, like photo-acoustics and magnetic resonance. This presentation is focused on temperature measurements taken by magnetic resonance. The temperature variation during the thermal treatment can be directly measured with magnetic resonance images, by using the so-called PRF-Shift method (PRF is the acronym for Proton Resonance Frequency). On the other hand, the classical linear relation between the temperature variation and the phase-shift can be used as an observation model, together with a bioheat transfer evolution model, for the solution of state estimation problems with Bayesian filters. By using synthetic phase-shift measurements containing random errors, a steady-state version of the Kalman filter was applied to recover the transient temperature of the tissues itself, besides the temperature variation directly provided by the magnetic resonance measurements. In addition, the solution of the state estimation problem allowed for an increase in the resolution of the thermal image obtained with the classical PRF-Shift thermometry method. This presentation also addresses the solution of inverse problems with actual measurements during thermal ablation procedures of an agar-gel phantom, in order to reduce effects of artefacts in the PRF-Shift thermometry images. These artefacts appeared in the vicinity of the intrusive radio frequency electrode, where the thermal damage was more significant due to the highest temperatures. Temperature measurements provided by the PRF-Shift thermometry in regions not affected by the electrode were used for the solution of the inverse heat transfer problem, thus allowing for the indirect observation of the temperatures in the region of interest and their associated uncertainties.

Recent progress on the inverse scattering theory for ideal Alfvén waves

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Keywords: Magnetohydrodynamics; Alfvén Wave; Inverse Scattering; Energy Method

The Alfvén waves are fundamental wave phenomena in magnetized plasmas. Mathematically, the dynamics of Alfvén waves are governed by a system of nonlinear partial differential equations called the magnetohydrodynamics (MHD) equations. Let us introduce some recent results about inverse scattering of Alfvén waves in ideal MHD, which are intended to establish the relationship between Alfvén waves emanating from the plasma and their scattering fields at infinities. The proof is mainly based on the weighted energy estimates. Moreover, the null structure inherent in MHD equations is thoroughly examined, especially when we estimate the pressure term.

Bayesian parameter identification in the Landau-de Gennes theory for nematic liquid crystals

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Keywords: Landau-de Gennes model, nematic liquid crystals, semilinear elliptic problems, statistical inverse problems, Markov chain Monte Carlo

Liquid crystals are anisotropic electro-optical materials with characteristics of both liquid and crystalline phases. Widely known for their use in display technology, recent advances have expanded their applications to include biological sensors, soft robotics, smart windows, and advanced optical devices. In this talk, we discuss a pathway to reconstruct material parameters from measurements within the Landau-de Gennes model for nematic liquid crystals. We present a Bayesian approach to this inverse problem and analyse its properties using given, simulated data for benchmark problems of a planarbistable nematic device. In particular, we discuss the accuracy of the Markov chain Monte Carlo approximations, confidence intervals and the limits of identifiability.

On optimality and bounds for internal solutions generated from impedance data driven Gramians.

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Keywords: inverse scattering; reduced order models, internal solutions

We consider the computation of internal solutions for a time domain plasma wave equation with unknown coefficients from the data obtained by sampling its transfer function at the boundary. The computation is performed by transforming the background snapshots for a known background coefficient using the Cholesky decomposition of the data-driven Gramian. We show that this approximation is asymptotically close to the projection of the internal solution onto the subspace of background snapshots. This allows us to derive a generally applicable bound for the error in the approximation of internal fields from boundary data only for a time domain plasma wave equation with an unknown potential q. We use this to show convergence for general unknown q in one dimension. We show numerical experiments and applications to SAR imaging in higher dimensions.

Neural network parametrized level sets for segmentation

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Keywords: Neural network; Segmentation; Level set method

Level set methods are a well-established approach in image segmentation, with the ChanVese multiphase model being a notable example [1]. Recently, some researchers have explored parameterized versions of level set methods to enhance segmentation quality [2]. Meanwhile, the rapid growth of neural networks in imaging has led to powerful data-driven segmentation tools. In this talk, we explore a mathematical connection between the parameterized ChanVese model and neural networks. The aim is to understand how classical variational models relate to neural network formulations. Specifically, we prove that a parameterized multiphase level set function corresponds to a two-layer neural network. This result provides a mathematical interpretation of certain neural network structures used in imaging tasks. Furthermore, numerical experiments are presented to demonstrate the feasibility of the proposed approach.

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A Novel Approach for Solving HamiltonJacobi Equations with Applications to Optimal Transport

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This talk presents a novel solution formula for effectively solving initial value problems of HamiltonJacobi partial differential equations (HJ PDEs). Although HJ PDEs are fundamental in many applications, the non-uniqueness and non-smoothness of solutions present significant challenges in obtaining viscosity solutions. We introduce an implicit solution formula derived from the method of characteristics and explore its connection with classical Hopf-type formulas. Building on this formulation, we propose a deep learning-based methodology for solving HJ PDEs without relying on supervised data. By leveraging the mesh-free nature of neural networks, the method offers a scalable, efficient, and accurate framework for addressing high-dimensional and even nonconvex problems. Furthermore, we demonstrate the broad applicability and flexibility of the proposed formulation by extending it to problems in optimal transport.

A one-cut conditional gradient method for total variation regularization

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Keywords: Total Variation Regularization; Nonsmooth Optimization; Sparsity

A common ansatz in inverse problems for PDEs is that the sought solutions are piecewise constant, modelling situations like localized inclusions of different material properties within an otherwise homogeneous medium. In this situation, variational regularization with a total variation penalty balances being compatible with piecewise constant minimizers with retaining convexity of the regularizer. However, its lack of differentiability means that most numerical methods require some level of smoothing, so that such piecewise constant structures can be observed only approximately and/or at very fine resolutions.

In this talk, we instead focus on a fully-corrective generalized conditional gradient method alternating between updating an active set of subsets of the spatial domain and an iterate given by a conic combination of the associated characteristic functions. Different to previous approaches in the same spirit, the computation of a new candidate set only requires the solution of one prescribed mean curvature problem instead of the resolution of a fractional minimization task analogous to finding a generalized Cheeger set. After discretization on a triangulation used for FEM simulations for the forward operator, the former can be realized by a single run of a graph cut algorithm. This method allows to prove not just a global sublinear convergence under mild assumptions, but also its asymptotic linear convergence in more restrictive continuum settings using results of stability of surfaces of prescribed mean curvature under perturbations of the curvature.

This talk is based on the papers [1, 2].

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Quantitative magnetic resonance imaging: data-driven, physics integrated models

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Keywords: Quantitative imaging; MRI; hybrid data-driven models, machine learning

Inspired by applications in physics-integrated magnetic resonance imaging resp. fingerprinting, differential equation constrained optimization problems with constituents that are only accessible through data-driven techniques are studied. A particular focus is on the analysis and on numerical methods for problems with machine-learned components. For a rather general context, an error analysis is provided, and particular properties resulting from artificial neural network based approximations are addressed. Moreover, for the MRI application analytical details are presented and numerical results are provided.

Range description for the free space wave operator

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Keywords: Spherical means transform, range conditions, wave operator, PAT/TAT

The forward problem arising in several hybrid imaging modalities can be modeled by the Cauchy problem for the free space wave equation. Solution to this problems describes propagation of a pressure wave, generated by a source supported inside unit sphere S. The data g represent the time-dependent values of the pressure on the observation surface S. Finding initial pressure f from the known values of g consitutes the inverse problem. The latter is also frequently formulated in terms of the spherical means of f with centers on S.

We consider a problem of range description of the wave operator mapping f into g. Such a problem was considered before, with data g known on time interval at least $[0, 2]$ (assuming the unit speed of sound). Range conditions were also found in terms of spherical means, with radii of integration spheres lying in the range $[0, 2]$. However, such data are redundant. We present necessary and sufficient conditions for function g to be in the range of the wave operator, for g given on a half-time interval $[0, 1]$. This also implies range conditions on spherical means measured for the radii in the range $[0, 1]$.

A Fully Stochastic Reconstruction Method for Quantitative Photoacoustic Tomography

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Keywords: Radiative Transfer Equation; Monte Carlo; Stochastic Optimisation; Photoacoustic Tomography

Coupled Physics Imaging methods combine image contrast from one physical process with observations using a secondary process; several modalities in opto-acousto imaging follow this concept wherein optical contrast is observed with acoustic measurements. For the inverse problem both an optical and acoustic model need to be inverted.

In tomographic problems based on photon propagation, the deterministic model of choice is The Radiative Transport Equation (RTE). Whereas it has a direct solution for non-scattering media it is complicated to solve for cases involving significant scattering. In these cases, Monte Carlo (MC) methods are a widely applicable and accurate class of stochastic modelling techniques that converge to the deterministic solution in the limit of an infinite number simulated photons.

Classical methods that involve a non-linear optimisation approach can be combined with advances in stochastic subsamplings strategies that are in part inspired by machine learning applications. In such approaches the forward problem is considered deterministic and the stochasticity involves splitting of an objective function into sub functions that approach the fully sampled problem in an expectation sense.

In this work we conside a setting where the forward and inverse problems are both solved stochastically. By adjusting the batch size in the forward and inverse problems together, we can achieve better performance than if subsampling is performed seperately.

Estimation of electrical conductivity and permittivity in quantitative thermoacoustic tomography

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Keywords: Quantitative thermoacoustic tomography, Bayesian methods, Maxwell's equations, *maximum a posteriori*, optimisation

In this work, we study the inverse problem of quantitative thermoacoustic tomography (QTAT). In QTAT, a short pulse of micro- or radio waves is directed to the imaged target. As the electromagnetic waves propagate within the target, they are absorbed by various molecules, resulting in an absorbed energy density. In the inverse problem of QTAT, the dielectric parameters, such as electrical conductivity and permittivity, of the imaged target are estimated from the absorbed energy density.

We propose an approach for simultaneous estimation of the electrical parameters in QTAT. The inverse problem is approached in the framework of Bayesian inverse problems. We compute *maximum a posteriori* estimates and study their reliability using the Laplaces approximation of the posterior distribution. The forward model is based on Maxwells equations, and its solution is approximated numerically using the finite element method. The approach is studied using numerical simulations with one and two electromagnetic excitations at multiple excitation frequencies. The results show that the dielectric parameters can be estimated using the proposed approach. However, the problem can suffer from non-uniqueness when only one electromagnetic excitation is used.

Large-Scale Model-Based 3D Image Reconstruction for Raster-Scan Optoacoustic Mesoscopy

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Keywords: Large-Scale Inverse Problem; Stochastic Optimization; Image Reconstruction; Variational Regularization; Optoacoustic Imaging

Optoacoustic mesoscopy is a biomedical imaging technique that provides images of the skin's microvasculature. Skin biomarkers derived from these images have the potential to aid in the diagnosis and monitoring of diseases such as diabetes, psoriasis, and cancer. The data acquisition process starts with the illumination of the skin using a short laser light pulse. The laser light is absorbed by hemoglobin in the blood vessels. The absorbed light energy is converted into heat energy, leading to a temperature rise that results in local volumetric expansion and contraction, ultimately generating pressure waves. The pressure waves originating from the blood vessels propagate through the skin and a coupling medium and are detected by an ultrasound transducer that measures the pressure waves as an electrical signal. From this electrical signal, one can reconstruct an image of the blood vessels.

In this talk we focus on a spherically focused ultrasound transducer performing a raster-scan on the skin surface and use a model-based approach to reconstruct 3D images from the signal measurements. The model-based reconstruction approach consists of computing the forward model matrix that describes the measurement process and using variational regularization to solve an optimization problem incorporating the forward model. A main challenge of the model-based reconstruction approach for 3D images is the excessive memory demand and the computation time. We reduce both by exploiting the symmetry of the ultrasound transducer and using a stochastic proximal gradient algorithm for iterative image reconstruction. In each iteration we randomly choose a certain number of model matrix rows and corresponding signal values for which we perform a gradient descent step on the data fidelity term and a proximal step on the regularization term using the primal-dual hybrid gradient algorithm.

FFT-Accelerated Inverse Volume Scattering in Three Dimensions Using Continuation in Frequency

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Keywords: Inverse Scattering; Continuation in Frequency

In this work, we present a fast, robust, and accurate algorithm for solving an inverse problem aimed at reconstructing the sound speed profile of a three-dimensional variable medium from multifrequency scattered data. The inverse problem is formulated as a collection of PDE-constrained optimization problems, one for each frequency of interest. To enhance stability and reduce computational cost compared to solving the full multifrequency optimization problem directly, our solver employs a continuation-in-frequency strategy [1], which sequentially solves single-frequency inverse scattering problems in order of increasing frequency using the steepest descent method.

Each single-frequency problem is inherently ill-posed and nonlinear. The ill-posedness is mitigated by constraining the solution to a band-limited representation of the sound speed profile, while the nonlinearity is addressed through an iterative optimization approach. To further accelerate the overall computation, the forward volume scattering problems are solved using an FFT-based method developed by Vico et al. [2].

We provide numerical experiments that demonstrate the effectiveness of our algorithm in reconstructing both obstacles and smoothly varying sound speed profiles.

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Fast accelerated solver for the inverse obstacle scattering using multifrequency data

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Keywords: Inverse Scattering; Continuation in Frequency

In this work, we consider the problem of reconstructing the shape of an impenetrable sound-soft obstacle from measurements of the scattered field generated by an incident plane wave striking the obstacle from multiple directions and at various frequencies. This inverse scattering problem is inherently ill-posed, nonlinear, and computationally demanding. We introduce a comprehensive framework designed to effectively overcome these challenges.

Following the approach in [1], we reformulate the inverse problem as a PDE-constrained optimization problem. To reduce computational cost, we employ a continuation-in-frequency strategy [2], solving a sequence of single-frequency inverse problems in order of increasing frequency. Each of these single-frequency problems is solved using an iterative optimization method.

To accelerate the forward solver required at each iteration, we employ the chunkIE toolbox [3], which offers fast and accurate solvers for boundary integral equations. The ill-posedness of the inverse problem is addressed by implicitly representing the obstacle as the level curve of a potential function. We regularize the reconstruction using bandlimited updates to the domain and ensure geometric stability across iterations with a smoother mesh generator, following the method in [4].

We present numerical experiments that demonstrate the robustness and effectiveness of the proposed framework in accurately recovering complex obstacle geometries.

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Numerical Methods for a Fractional Helmholtz Problem

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Keywords: Inverse scattering; Fractional PDE; Inverse Born Series

This talk discusses a recent scattering problem for the fractional Helmholtz equation. Fast methods will be applied to both the forward and inverse problems, including a new application of the inverse Born series. The talk concludes with a numerical comparison to a standard inverse scattering problem.

A fast direct solver for PDEs on surfaces

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Keywords: Laplace-Beltrami equation; Fast direct solvers; High order discretizations

Elliptic PDEs on a surface embedded in three dimensions occur in many forward scattering problems. For example, they are used in the study of surface wave phenomenon and in electromagnetic scattering problems, where they are enable to computation of Hodge decompositions of surface currents. PDEs on surfaces are also frequently used in computer graphics for shape analysis and surface interpolation problems.

In this talk, we present a method for converting a broad class of scalar elliptic PDEs on a general smooth surface into second kind Fredholm integral equations. Doing so ensures the equations are well conditioned and makes it possible to construct high-order numerical solvers. To derive the integral equation, we extend the known method for the Laplace-Beltrami problem [1] on a sphere to a broader class of equations on general smooth surfaces. Specifically, we observe that the Greens function of a corresponding PDE in the plane gives a parametrix (an approximate Greens function) for the PDE on a surface. We then use that parametrix to derive an integral equation form of the PDE.

We will also discuss how the structure and simplicity of the resulting integral equation can be leveraged to build a fast direct solver for the discretized linear system. This fast direct solver is based on the recursive skeletonization method presented in [2]. These methods are usually developed for kernels that satisfy a Green's theorem. Since ours do not, we shall prove that a proxy shell technique can be combined with recursive skeletonization to build an accurate fast direct solver. If time allows, we will also discuss simple, high order quadrature formulas based on the techniques in [3] that allow us to easily discretize the resulting integral equation.

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Compressed sensing for photoacoustic tomography

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Keywords: Photoacoustic tomography, compressed sensing.

Photoacoustic tomography is an emerging medical imaging technology whose primary aim is to map the high-contrast optical properties of biological tissues by leveraging high-resolution ultrasound measurements. Mathematically, this can be framed as an inverse source problem for the wave equation; the pressure wave u is generated by an initial pressure $u_0 \in L^2(\mathbb{R}^3)$, which is supported within a compact set $K \subset \mathbb{R}^3$, and the wave dynamics is described by the associated initial value problem:

$$\begin{cases} u_{tt} - c^2 \Delta u = 0, & \text{in } \mathbb{R}^3 \times [0, +\infty), \\ u(\cdot, 0) = u_0, & \text{in } \mathbb{R}^3, \\ u_t(\cdot, 0) = 0, & \text{in } \mathbb{R}^3. \end{cases} \tag{1}$$

The pressure wave u is measured on a (smooth) acquisition surface $\Sigma \subset \mathbb{R}^3$ over a finite time interval $[0, T]$, with $T < +\infty$.

In this work, for the first time, it is shown how, by assuming signal sparsity, it is possible to establish rigorous stable recovery guarantees when the data collection is given by spatial averages restricted to a limited portion of the boundary. Our framework encompasses many approaches that have been considered in the literature. The result is a consequence of a general framework for subsampled inverse problems developed in previous works and refined stability estimates for an inverse problem for the wave equation with surface measurements.

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Stochastic algorithms for the Computed Tomography (CT) problems

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Keywords: First order stochastic gradient methods; Machine Learning; Computed Tomography

A discrete linear inverse problem is generally formulated as $y = Kx + \varepsilon$ where $x \in \mathbb{R}^n$ is the quantity or object to be reconstructed (ground truth), $K \in \mathbb{R}^{m\times n}$ is the discretization of the operator modeling the physical problem, $y \in \mathbb{R}^m$ represents the measured data, and $\varepsilon \in \mathbb{R}^m$ denotes the Gaussian noise affecting the data [1]. In the case of Computed Tomography (CT), K is given by the Radon transform. In the discrete setting, the problem exhibits ill-conditioning, making it unsuitable for direct inversion. One standard approach to tackle this issue is variational optimization [2], which transforms the inverse problem into the following minimization problem

$$\min_{x\in\mathbb{R}^d} D(Kx, y) + \beta R(x)$$

where D is a discrepancy function of Kx from the measured data, $R(x)$ is a regularization term, which controls the influence of noise on the solution and preserves important features of the reconstruction, and β balances the contribution of the two terms (regularization parameter).
In recent decades, first-order deterministic algorithms have been widely used to solve such variational problems due to their efficiency, simplicity, and relatively fast convergence [3, 5]. However, as data complexity and size have increased, the computational costs of these methods have become prohibitive. To address this, the focus of this talk has shifted towards stochastic optimization. Stochastic algorithms, commonly used in machine learning to solve large-scale problems, can efficiently handle large datasets by using only a subset of the available data at each iteration. This reduces computational costs per iteration while ensuring, under suitable assumptions, convergence to the solution [4].
The aim of this talk is to propose a strategy for employing stochastic methods in the context of CT problems that adaptively manage the hyperparameters on which their efficiency depends. These methods, originally developed for machine learning and deep learning [6, 7], are adapted to the imaging context by analyzing the processes of the mini-batch sampling, step-size selection strategies, and simultaneously by preservation the convergence assumptions. Numerical simulations will be used to validate the proposed approach.

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A convex lifting approach for the Calderón problem

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Keywords: Calderón problem; Lifting; Convex relaxation.

The Calderón problem [1] concerns the recovery of an unknown coefficient of a partial differential equation from the boundary values of its solution. These measurements induce a highly nonlinear forward operator, posing challenges for the development of reconstruction methods, which usually suffer from the problem of local convergence. To circumvent this issue, we propose an alternative approach based on lifting and convex relaxation techniques, that has been successfully developed for finite-dimensional quadratic inverse problems [2, 4, 5]. This leads to a convex optimization problem whose solution coincides with the sought-after coefficient, provided that a non-degenerate source condition holds [3, 6]. While we leave the analysis of the source condition in the Calderón setting to future works, we demonstrate the validity of our approach in a toy model where the solution of the PDE is known everywhere in the domain (instead of only voltage-to-current measurements on the boundary). In this simplified setting, we verify that the non-degenerate source condition holds under certain assumptions on the unknown coefficient.

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Learning Variational Models: From Bilevel Optimization to Algorithm Unrolling

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Keywords: Computational Imaging; Optimization; Algorithm Unfolding

Variational models have long been a standard tool for the resolution of linear inverse problems. In recent times, especially with the rise of deep learning methodologies, their flexibility has been questioned. This talk will explore two possibilities to improve on the classical variational framework in order to overcome some of its limitations. More specifically, exploiting machine learning techniques allows for the use of more complex and more general models. Bilevel optimization cast, as the name suggests, two nested optimization problems. The inner one is the one dedicated to finding the minimum of the variational model, while the outer one focuses on selecting the best parameter configuration for it. Solving bilevel optimization problems is not straightforward, as the minimum of the inner problem is rarely available explicitly. This calls for the use of optimization algorithms that are able to deal with inexactness [2]. Algorithm unrolling [2] tries to circumvent this issue by fixing a computational budget, that is, it fixes a number of iterations for the solver of the inner problem. In this case, the resulting scheme is that of a simple neural network, whose structure is guided by the chosen solver. These hybrid strategies, compared to more pure deep learning architectures, are able to exploit the interpretability of variational models, while still improving the overall results.

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A bilevel learning framework for variational image reconstruction with learned convex regularizer

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Keywords: variational problem; learned convex regularizer; bilevel Learning; primal-dual algorithm

In inverse problems, a classical reconstruction framework is based on the idea of variational regularization. This approach encodes prior information about potential reconstructions through a variational regularizer. Traditional methods often employ hand-crafted regularizers, such as total variation (TV) and total generalized variation (TGV). With the success of deep learning image reconstruction, there has been increasing interest in data-driven regularizers, potentially replacing manually designed regularizers with neural networks. In this talk, we explore learned convex regularizers within the variational setting, specifically when the convex regularizer is parameterized by an Input Convex Neural Network (ICNN). While various attempts have been made to learn regularizers, the adversarial training approach utilizes a loss that aims to separate ground-truth images from those with artifacts. However, the adversarial regularizer is not explicitly trained for the reconstruction task. To address this, we present a bilevel optimization framework to incorporate reconstruction quality in the learning process. We will also present a novel approach of solving the variational problem within a primal-dual framework. By introducing auxiliary variables corresponding to the activations of intermediate layers, we eliminate the nested nature of a neural network. This reformulates the variational problem as a constrained convex problem.

Unfolded proximal neural networks for computational imaging

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Keywords: Image denoising, Image restoration, Unrolled proximal algorithms, Unfolded neural networks

A common approach to solve inverse imaging problems relies on finding a maximum a posteriori (MAP) estimate of the original unknown image, by solving a minimization problem. In this context, iterative proximal algorithms are widely used, enabling to handle non-smooth functions and linear operators. Recently, these algorithms have been paired with deep learning strategies, to further improve the estimate quality. In particular, proximal neural networks (PNNs) have been introduced, obtained by unrolling a proximal algorithm as for finding a MAP estimate, but over a fixed number of iterations, with learned linear operators and parameters. As PNNs are based on optimization theory, they are very flexible, and can be adapted to any image restoration task, as soon as a proximal algorithm can solve it. They further have much lighter architectures than traditional networks.

In this presentation we describe a unified framework to build PNNs for the Gaussian denoising task, based on both the dual-FB and the primal-dual Chambolle-Pock algorithms. We further show that accelerated inertial versions of these algorithms enable skip connections in the associated NN layers. We propose different learning strategies for our PNN framework, and investigate their robustness (Lipschitz property) and denoising efficiency. Finally, we show that the proposed PNNs can be either plugged in a forward-backward algorithm for an image deblurring problem, or used for edge detection.

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Learning (simple) regularizers for inverse problems

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Keywords: Inverse Problems, Learned Regularization, Sparsity, ℓ^1 regularization

In this talk, we consider the problem of learning the optimal regularizer for linear inverse problems modeled in separable Hilbert spaces. In the context of generalized Tikhonov regularization, we characterize the optimal regularizer and derive generalization estimates, in both supervised and unsupervised settings. In the context of sparsity promoting regularization, we derive generalization estimates for learning the optimal change of basis in the ℓ^1 penalty term.

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Bilevel Hyperparameter Learning for Nonsmooth Regularized Imaging and Machine Learning Models

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Keywords: Bilevel Optimization; Variational Analysis, Nonsmooth Analysis, Sparse Machine Learning Models

We study a bilevel optimization framework for hyperparameter tuning of variational models, with a focus on sparse regression and classification tasks. In particular, we consider a weighted elastic-net regularizer, where feature-wise regularization parameters are learned through a bilevel formulation. A key novelty of our approach is the use of the Forward-Backward Envelope (FBE) to smooth the nonsmooth lower-level problem while preserving its set of minimizers. This reformulation yields a bilevel objective composed with a locally Lipschitz solution map, allowing the application of generalized subdifferential calculus to derive basic subgradients and enable efficient subgradient-based optimization. Empirical results on synthetic datasets demonstrate that our approach significantly outperforms scalar regularization methods in terms of prediction accuracy and support recovery. These findings highlight the benefits of feature-wise regularization and the effectiveness of bilevel optimization as a principled framework for learning interpretable and high-performing models.

Math Meets Genomics: Using Optimization to Track Pathogen Variants

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Keywords: Mixed-Integer Programming; Strain Reconstruction

Understanding which genetic variants (or ”strains”) of a pathogen are present in a biological sample is important for studying how diseases spread, evolve, and respond to treatments. However, when a sample contains a mixture of multiple strains, it becomes difficult to figure out exactly which strains are present and in what proportions. This is a common challenge in public health.

To address this, lab techniques are employed to produce a vector of mutation frequencies at specific sites in the genome. Think of this vector as a kind of summary of the genetic content in the mixed sample we know how common each mutation is, but we dont know which mutations go together in the same strain.

The main mathematical problem we tackle is an inverse problem: given this vector of observed mutation frequencies, can we reconstruct the original strain sequences and determine how much of each strain is present in the mixture?

We formulate this as a Mixed-Integer Quadratic Programming (MIQP) problem. The MIQP model searches for a set of discrete strain sequences (combinations of mutations) and their continuous proportions (how much of each strain is present) that together best explain the observed mutation frequencies. We’ll describe how this model is constructed and solved, and well show results from computational experiments. The results suggest that our approach can be a useful tool for solving practical inverse problems in disease surveillance.

Physics-Informed Neural Networks for Water Flow Prediction and Flood Control in a Watershed

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Keywords: Inverse Problem; Physics–Informed Neural Networks; Hydrological Forecasting, Parameter Identification; Model Predictive Control; Flood Mitigation.

The research addresses the problem of flood mitigation in a watershed through an integrated modeling and control framework, formulated as an inverse problem. The proposed approach uses Physics-Informed Neural Networks (PINNs) to model the water flow process and efficiently estimate unknown physical parameters in the Saint-Venant equations, which govern the flow dynamics in river channels. Using sparse and irregular observational data, the PINNs are trained to infer internal system states and adjust model parameters, thus solving a typical inverse problem. The training process is enhanced by a strategy inspired by Bayesian inference, which improves the robustness and reliability of parameter estimation. Once identified, PINN-based surrogate models are embedded within a Model Predictive Control (MPC) framework, enabling real-time dam gate control to minimize downstream flooding impacts. The results of simulations and real-world case studies demonstrate the effectiveness of the PINN-MPC methodology in both prediction accuracy and parameter identification, offering an innovative and computationally efficient alternative to conventional hydrodynamic modeling and control strategies.

Similarity Learning with Neural Networks

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Keywords: Self-Similarity; Renormalization; Machine Learning.

We present a neural network framework that performs an inverse problem by discovering similarity relations directly from raw data, thereby recovering the underlying physical laws governing dimensionless quantities. In tandem, we develop a linear algebra approach to identify the corresponding symmetry groups that encode these similarity relations. This dual methodologyfrom unstructured data to dimensionless lawsprovides a robust, data-driven avenue for inferring fundamental physics in complex systems.

Although broadly applicable, we demonstrate the approach in fluid mechanics, covering laminar Newtonian and non-Newtonian flows in smooth pipes, as well as turbulent flows in both smooth and rough pipes. These examples illustrate the methods versatility, handling both straightforward and highly intricate flow regimes, while confirming its effectiveness in uncovering key physical laws through the lens of similarity and symmetry.

Inverse Problem of Parameter Calibration from Diverse Observations for Complex Hydrocarbon Mixtures

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Keywords: Bayesian calibration; Gaussian processes; uncertainty quantification; inverse problems; data-driven modeling

We address the inverse problem of estimating binary interaction parameters (BIPs) in flash calculations for multicomponent hydrocarbon mixtures, a critical step in industrial applications such as flow assurance, reservoir simulation, and chemical process design. Traditional deterministic approaches rely on gradient-based optimization to match experimental data but often neglect uncertainties and unmodeled physics. In contrast, we adopt a Bayesian calibration framework that systematically accounts for observational and modeling errors.

A Gaussian process surrogate approximates the thermodynamic simulator, alleviating the computational cost of repeatedly solving a stiff, nonlinear system. We then use a second Gaussian process to capture unmodeled physical phenomena and measurement noise. Combining these components in a Markov Chain Monte Carlo scheme yields posterior distributions for both model parameters and observational errors. The result is a data-driven, adaptive approach that can update predictions as new observations emerge, enhancing reliability and decision-making in complex industrial processes through richer insight and improved uncertainty quantification compared to single-point deterministic estimates.

Viscous-inertial waves on the surface of the Sun: modeling, forward and inverse problems

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Keywords: helioseismology, inertial waves, stream function, parameter identification, iterative regularization

The recent discovery of inertial waves on the surface of the Sun offers new possibilities to learn about the solar interior. Inertial oscillations are caused by the Coriolis force, exhibiting periodic behavior on the timescale of the 27-day solar rotation cycle. In this talk, we discuss modeling, forward and inverse problems for the Sun via newly observed inertial waves. In particular, we derive the inertial oscillation using the stream function, turning the vectorial motion equation into a scalar equation in a low-dimensional setting. For the forward problem, we prove unique existence of the wave solution under small latitudinal variations of the rotation. For the inverse problem of viscosity and rotation identification, we investigate different observation scenarios with full and leaked data.

Variably Scaled Kernels for the regularized solution of the parametric Fourier imaging problem

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Keywords: Variably Scaled Kernels, Lebesgue functions, Inverse problems, Solar X-ray imaging

We address the problem of approximating parametric Fourier imaging problems via interpolation/extrapolation algorithms that impose smoothing constraints across contiguous values of the parameter. Previous works already proved that interpolating via Variably Scaled Kernels (VSKs) the scattered observations in the Fourier domain and then defining the sought approximation via the projected Landweber iterative scheme, turns out to be effective [1]. This study provides new theoretical insights, including error bounds in the image space and properties of the projected Landweber iterative scheme, both influenced by the choice of the scaling function, which characterizes the VSK basis. Such bounds then suggest a smarter solution for the definition of the scaling functions. Indeed, by means of VSKs, the information coded in an image reconstructed for a given parameter is transferred during the reconstruction process to a contiguous parameter value. Benchmark test cases in the field of astronomical imaging, numerically show that the proposed scheme is able to regularize along the parameter direction [2], thus proving reliable and interpretable results.

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Feature Understanding and Sparsity Enhancement via 2-Layered kernel machines (2L-FUSE)

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Keywords: Feature reduction, kernel models, regression learning

We propose a novel sparsity enhancement strategy for regression tasks, based on learning a data-adaptive kernel metric, i.e., a shape matrix, through 2-Layered kernel machines [2]. The resulting shape matrix, which defines a Mahalanobis-type deformation of the input space, is then decomposed via Singular Value Decomposition (SVD), allowing us to identify the most informative directions. This task-aware approach provides a flexible, interpretable, and accurate feature reduction scheme. Numerical experiments on both synthetic and real-world datasets demonstrate that our approach achieves minimal yet highly informative feature sets without loosing predictive performances. We further consider an application within the field of solar physics, focusing on the prediction of a geomagnetic storms [1].

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3D reconstruction of solar flare emission

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Keywords: Singular value decomposition (SVD); 3D reconstruction; solar flare imaging

Since April 2023, simultaneous observations of solar flares have become possible with Solar Orbiter/STIX (Spectrometer Telescope for Imaging X-rays, [2]) and ASO-S/HXI (Hard X-ray Imager, [1]), providing X-ray observations from two distinct viewpoints. The measurements from these instruments correspond to a sparse sampling of the Fourier transform of the photon emission density per unit volume. This unique observational setup enables, for the first time, the potential for three-dimensional reconstructions of solar flares. However, the inherent sparsity of the sampling makes this reconstruction problem ill-posed.

In this presentation, we characterize the forward operator describing this problem and analyze its singular value decomposition (SVD) [3, 4]. The SVD is expressed as a function of the relative angle between the positions of the two instruments and the location of the flare on the solar surface. This analysis allows us to quantify the ill-posedness of the problem and to determine conditions under which stable reconstructions are feasible.

Finally, we present preliminary three-dimensional reconstructions from real data of a solar flare event that occurred on October 3, 2024, between 12:00 and 13:00 UT.

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Variable Projection Methods for Large-scale Inverse Problems

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Keywords: variable projection; Tikhonov regularization; separable nonlinear least squares

We consider discrete ill-posed inverse problems of the form

$$\mathbf{A}(\mathbf{y})\mathbf{x} \approx \mathbf{b} = \mathbf{b}_{\text{true}} + \boldsymbol{\epsilon},$$

where $\mathbf{A}(\mathbf{y})$ is a parametrized ill-conditioned forward operator and the data vector $\mathbf{b}$ is corrupted by noise. The unknowns are $\mathbf{x}$ and the parameter vector $\mathbf{y}$ that defines $\mathbf{A}(\mathbf{y})$. Problems of this form arise in semi-blind deblurring and other applications where $\mathbf{A}$ is not fully known.

To compute a stable solution, we minimize a Tikhonov-regularized functional:

$$\min_{\mathbf{x},\mathbf{y}} \frac{1}{2}\|\mathbf{A}(\mathbf{y})\mathbf{x} - \mathbf{b}\|_2^2 + \frac{\lambda^2}{2}\|\mathbf{L}\mathbf{x}\|_2^2 + \mu\mathcal{R}(\mathbf{y}),$$

where $\mathbf{L}$ is a regularization operator and $\mathcal{R}(\mathbf{y})$ is a regularizer on $\mathbf{y}$. We discuss extensions of the Variable Projection (VarPro) framework to this setting. This includes GenVarPro for general-form regularization [1] and Inexact-GenVarPro [2], which uses iterative solvers such as LSQR and specific stopping criteria for local convergence.

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Parameter Recovery in Inverse Gravimetry and EIT from Limited Measurements

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Keywords: Gravimetry; EIT

This talk addresses two inverse problems involving limited data. First, we consider inverse gravimetry, where the goal is to recover the mass density distribution of a body from gravitational measurements. Unlike traditional approaches that rely on data collected over curves or surfaces, we focus on cases where only sparse pointwise measurements are available. Instead of reconstructing a full density distribution, we target the recovery of geometric parameters for simple shapes, such as ellipses and rectangles in 2D or ellipsoids and rectangular parallelepipeds in 3D. Second, we study Electrical Impedance Tomography (EIT), aiming to determine the conductivity distribution inside a domain Ω from boundary voltage and current measurements. Specifically, we investigate the identification of a small elliptical conductivity anomaly within a unit disc, assuming a slight perturbation from a constant background. Our approach introduces movable electrode pairs for additional measurements, proving that three measurements uniquely and stably recover the anomaly's location and size, while two more measurements resolve its aspect ratio and orientation. We also analyze the stability of the inverse problem and optimal experiment design.

Improved impedance inversion by iterated graph Laplacian

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Keywords: graph Laplacian; data adaptive; seismic inversion

Deep learning techniques have shown significant potential in many applications through recent years. The achieved results often outperform traditional techniques. However, the quality of a neural network highly depends on the used training data. Noisy, insufficient, or biased training data leads to sub-optimal results.

We present a hybrid method that combines deep learning with iterated graph Laplacian and show its application in acoustic impedance inversion which is a routine procedure in seismic explorations. A neural network is used to obtain a first approximation of the underlying acoustic impedance and construct a graph Laplacian matrix from this approximation. Afterwards, we use a Tikhonov-like variational method to solve the impedance inversion problem where the regularizer is based on the constructed graph Laplacian. The obtained solution can be shown to be more accurate and stable with respect to noise than the initial guess obtained by the neural network. This process can be iterated several times, each time constructing a new graph Laplacian matrix from the most recent reconstruction. The method converges after only a few iterations returning a much more accurate reconstruction.

We demonstrate the potential of our method on two different datasets and under various levels of noise. We use two different neural networks that have been introduced in previous works. The experiments show that our approach improves the reconstruction quality in the presence of noise.

A Preconditioned Version of a Nested Primal-Dual Algorithm for Image Deblurring

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Keywords: Ill-posed problems; Image deblurring; Convex optimization; Preconditioning

Regularization techniques for image deblurring problems typically consist of a smooth term and a potentially non-smooth convex term. A common approach to solving these problems is using proximal gradient methods. To accelerate the convergence of these first-order iterative algorithms, strategies such as variable metric methods have been introduced in the literature.

In particular, regarding the variable metric strategy proposed in [2], we proved that, for linear inverse problems, this approach can be reinterpreted as a right preconditioning method. Consequently, we explore an inexact left-preconditioned version of the same proximal gradient method. We prove the convergence of the new iteration to the minimum of a model where the norm of the data fidelity term depends on the preconditioner. The numerical results show that left and right preconditioning are comparable in terms of the number of iterations required to reach a prescribed tolerance, but left preconditioning needs much less CPU time, as it involves fewer evaluations of the preconditioner matrix compared to right preconditioning. The quality of the computed solutions with left and right preconditioning are comparable. Finally, we propose some non-stationary sequences of preconditioners that allow for fast and stable convergence to the solution of the variational problem with the classical ℓ^2–norm on the fidelity term.

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A Learned Inverse Problem formulation for medical image enhancement under Contrast Agent Reduction

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Keywords: Hybrid Methods; Neural Network Operator; Regularized Inverse Problem; Medical Image Computing

The use of contrast agents is pivotal in medical imaging for improving diagnostic accuracy, yet it comes with limitations, including rare adverse reactions, environmental concerns, and economic costs. This talk addresses the *contrast agent reduction* (CAR) problem: reducing the administered dose while preserving image quality.

While most deep learning approaches treat this task as a direct image-to-image translation using black-box end-to-end models, we propose a different paradigm rooted in inverse problem theory. We introduce a *learned inverse problem* (LIP) framework, where a neural network is trained not to directly solve the task, but to approximate the forward operator (mapping high-dose to low-dose images). This enables us to recast CAR as an inverse problem, solved via regularized optimization, allowing explicit control over data fidelity and prior information. By embedding the learned operator within a mathematically principled reconstruction scheme, we bridge the gap between data-driven models and physics-informed reconstruction, moving beyond purely empirical solutions.

Experiments on pre-clinical imaging data show improved accuracy and robustness compared to standard deep learning approaches, highlighting the value of integrating learning with the structure of inverse problems.

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Priorconditioned Sparsity-Promoting Projection Methods for Deterministic and Bayesian Linear Inverse Problems

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Keywords: Prior conditioning, generalized Krylov subspace, sparsity, majorization minimization, generalized sparse Bayesian learning

High-quality reconstructions of signals and images with sharp edges are needed in a wide range of applications. To overcome the large dimensionality of the parameter space and the complexity of the regularization functional, sparisty-promoting techniques for both deterministic and hierarchical Bayesian regularization rely on solving a sequence of high-dimensional iteratively reweighted least squares (IRLS) problems on a lower-dimensional subspace. Generalized Krylov subspace (GKS) methods are a particularly potent class of hybrid Krylov schemes that efficiently solve sequences of IRLS problems by projecting large-scale problems into a relatively small subspace and successively enlarging it. We refer to methods that promote sparsity and use GKS as S-GKS. A disadvantage of S-GKS methods is their slow convergence. In this talk, we discuss techniques that improve the convergence of S-GKS methods by combining them with priorconditioning, which we refer to as PS-GKS. Specifically, integrating the PS-GKS method into the IAS algorithm allows us to automatically select the shape/rate parameter of the involved generalized gamma hyper-prior, which is often fine-tuned otherwise. Furthermore, we proposed and investigated variations of the proposed PS-GKS method, including restarting and recycling (resPS-GKS and recPS-GKS). These respectively leverage restarted and recycled subspaces to overcome situations when memory limitations of storing the basis vectors are a concern. We provide a thorough theoretical analysis showing the benefits of priorconditioning for sparsity-promoting inverse problems. Numerical experiment are used to illustrate that the proposed PS-GKS method and its variants are competitive with or outperform other existing hybrid Krylov methods.

Vertex characterization via second-order topological derivatives

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Keywords: computer vision, topological asymptotic analysis, vertex characterization

This talk focuses on identifying vertex characteristics in 2D images using topological asymptotic analysis. Vertex characteristics include both the location and the type of the vertex, with the latter defined by the number of lines forming it and the corresponding angles. This problem is crucial for computer vision tasks, such as distinguishing between fore- and background objects in 3D scenes. We compute the second-order topological derivative of a Mumford-Shah type functional with respect to inclusion shapes representing various vertex types. This derivative assigns a likelihood to each pixel that a particular vertex type appears there. Numerical tests demonstrate the effectiveness of the proposed approach.

Unique continuation and stability estimates for inverse boundary value problems

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Keywords: Stibility estimates for inverse problems; Quantitative estimates of unique continuation

Abstract: Inverse boundary value problems involve the retrieval of unknown parameters of a partial differential equation (PDE) from boundary data. In practical scenarios, this entails reconstructing internal properties of a medium (e.g., conduction, stiffness, density) based on observations made at its boundary. Typically, parameter estimation problems are ill-posed according to Hadamard's definition: small errors in the data may result in uncontrollable errors in the unknowns. In view of the many applications, this leads to the search for appropriate methods to contain such instability. In this talk I will introduce the use of quantitative estimates of unique continuation for the study of stability estimates.

Concentration inequalities and signal recovery

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Keywords: Concentration inequality, stability, localization operator.

If we do not know the low frequencies of a function f, we can still recover the complete f in a stable way under the condition that the measure of supp(f) satisfies some bound depending on the interval of missed low frequencies. This follows from the Donoho-Stark result on the concentration of band-limited L^2 functions (conjectured in [**?**] and partially proved in [**?**] for sufficiently small subsets).

A natural finite-dimensional analogous problem is to replace the band-limited functions with polynomials of bounded degree. In this case, the concentration among all measurable sets of a given measure is maximum when the set is a disc and the polynomial is a constant ([**?**], [**?**]). In this talk, we will review these results in connection with signal recovery and we will see the stability of the concentration inequality.

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A piecewise constant levelset approach for semi-blind deconvolution: Application to barcode decoding

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Keywords: Semi-blind deconvolution; Barcode decoding; Piecewise constant levelset method

We consider the semi-blind deconvolution problem modelling the decoding of linear barcodes. The Piecewise Constant LevelSet (PCLS) ansatz in [2013, De Cezaro et al., Inv. Probl. **29** 015003] is used as starting point to propose and analyze an iterative method for solving the underlying inverse problem.

A proximal point approach for Plug-and-Play regularization with deep equilibrium networks

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Keywords: Plug-and-Play regularization, deep equilibrium networks, preconditioned proximal point algorithm

In the talk, we discuss a novel class of Plug-and-Play (PnP) regularization strategies for linear inverse problems. Generally, PnP regularization is motivated by proximal algorithms associated with operator splitting, where the operator that corresponds to the discrepancy part of the inverse problem is treated as usual whereas the resolvent of the operator that corresponds to the regularization is replaced by a black-box denoiser such as a deep neural network. We propose to replace this black box by an implicit operator that mimics the behavior of monotone operator deep equilibrium networks. This way, it is possible to derive an all-at-once formulation that incorporates the PnP-regularized solution of the inverse problem as well as the implicit equations associated with the deep equilibrium network. In particular, within an appropriate version of a preconditioned proximal point algorithm, popular building blocks for deep neural networks such as convolutional layers and non-smooth activation functions can be incorporated. We discuss convergence of the algorithm as well as training strategies based on implicit differentiation. Finally, exemplary networks and inverse problems are discussed in terms of numerical experiments.

The second step in hybrid inverse problems in limited view

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Keywords: acousto-electric tomography, non-vanishing Jacobian

Hybrid inverse problems combine two imaging modalities in order to make the reconstruction procedure more well-posed. They typically consist of two steps: One step to obtain internal data and another step to reconstruct the desired material parameter. In this talk I will focus on hybrid inverse problems that combine Electrical Impedance Tomography (EIT) with another imaging modality, ultrasound waves or magnetic resonance imaging, in order to reconstruct the electrical conductivity. Additionally, I consider a limited view setting, where one only has control over a part of the boundary. Relative to classical EIT these hybrid imaging techniques suffice with only two EIT measurements. However, the two boundary functions imposed on a part of the boundary for the EIT procedure should be chosen carefully, so that the corresponding internal data contains enough information for reconstruction of the conductivity. In this talk I will address under what conditions on the boundary functions this is the case, and I will go through a numerical example.

Acoustically Modulated Electromagnetic Inverse Source Problems

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Keywords: inverse source problem; multiwave imaging

The inverse source problem for the Maxwell equations is of fundamental interest and considerable appliedl importance, with applications ranging from geophysics to biomedical imaging. The problem is usually stated in the following form: determine the electric current density from boundary measurements of the electric and magnetic fields. It is well known that this problem is underdetermined and does not admit a unique solution. In this work we propose an alternative approach to the electromagnetic inverse source problem. In this approach the electric current density as well as the conductivity, electric permittivity and magnetic permeability are spatially modulated by an acoustic wave. In this manner, we find that it is possible to uniquely recover the current density from boundary measurements of the fields with Lipschitz stability. Numerical simulations are used to illustrate the analytical results.

Full-field Photoacoustic Tomography with Variable Sound Speed and Attenuation

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Keywords: Full-field; PAT

In the standard photoacoustic tomography (PAT) measurement setup, the data used consist of time-dependent signals measured on an observation surface. In contrast, the measurement data of the recently invented full-field detection technique provides the solution of the wave equation in the spatial domain at a single point in time. While reconstruction using classical PAT data has been extensively studied, not much is known about the full-field PAT problem. In this work, we study full-field photoacoustic tomography with spatially variable sound velocity and spatially variable attenuation. In particular, we reconstruct the initial pressures $p|_{t=0}$ and $p_t|_{t=0}$ from 2D projections of the full 3D acoustic pressure distribution at a given time.

Imaging Second Harmonic Generation in Quantitative Thermoacoustics

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Keywords: Quantitative thermoacoustics; second harmonic generation

We study an inverse problem for a coupled system of semilinear Helmholtz equations where we are interested in reconstructing multiple coefficients in the system from internal data measured in applications such as thermoacoustic imaging. We derive results on the uniqueness and stability of the inverse problem in the case of small boundary data based on the technique of first- and higher-order linearization. Numerical simulations are provided to illustrate the quality of reconstructions that can be expected from noisy data.

Nonparametric Instrumental Variable Regression through Stochastic Approximate Gradients

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Keywords: Statistical Inverse Problem, Non-parametric Instrumental Variable

Instrumental variables (IVs) provide a powerful strategy for identifying causal effects in the presence of unobservable confounders. Within the nonparametric setting (NPIV), recent methods have been based on nonlinear generalizations of Two-Stage Least Squares and on minimax formulations derived from moment conditions or duality. In a novel direction, we show how to formulate a functional stochastic gradient descent algorithm to tackle NPIV regression by directly minimizing the populational risk. We provide theoretical support in the form of bounds on the excess risk, and conduct numerical experiments showcasing our method's superior stability and competitive performance relative to current state-of-the-art alternatives. This algorithm enables flexible estimator choices, such as neural networks or kernel based methods, as well as non-quadratic loss functions, which may be suitable for structural equations beyond the setting of continuous outcomes and additive noise. Finally, we demonstrate this flexibility of our framework by presenting how it naturally addresses the important case of binary outcomes, which has received far less attention by recent developments in the NPIV literature.

Fitting Simultaneously the SPX and VIX Smiles

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Keywords: Volatility Estimation; Stochastic Models

In recent years, the mathematical finance community has been challenged by the problem of finding models that can fit simultaneously observed prices of vanilla (call and put) options on the S&P500 index and the volatility index (VIX). Many of the proposed models are non-Markovian, which may lead to rather involved theoretical analyses of the pricing problem and possibly complex numerical solutions. In contrast, we propose a Markovian approach, based on a stochastic volatility model with functional parameters. The corresponding forward Kolmogorov equation and its adjoint PDE are then used to evaluate option prices on the S&P500 and VIX indices. This approach simplifies the pricing problem, which is important for the estimation. However, there are still some important theoretical gaps that must be filled, which will be discussed.

Optimisation and simulation of portfolios with equities, futures and options

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Keywords: portfolio optimisation, asset allocation, options, futures contracts

There is a vast scientific literature on derivative pricing, especially for options, and an equally extensive body of work on portfolio optimisation. Literature integrating these asset classes into a unified framework, however, remains relatively scarce. Nonetheless, derivatives can significantly enhance portfolio performance by introducing non-trivial opportunities for hedging, leverage, and diversification that are not easily accessible through equities alone. In this work, we propose a practical approach to portfolio construction that incorporates all three asset classes. We discuss how to integrate them into portfolio optimisation models and into a simulation framework by considering the specific constraints and dynamics of both futures and options trading. We provide empirical evidence that combining these asset classes can lead to improved risk-adjusted performance and more robust portfolios.

A Statistical Learning Approach to Local Volatility Calibration and Option Pricing

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Keywords: Bayes Theorem, Maximum Entropy Density, Option Pricing, Local Volatility Model, Calibration

By combining Bayes' theorem and maximum entropy densities (MED), we propose an accurate and computationally efficient technique for European option pricing and local volatility calibration. The resulting data driven technique avoids the solution of partial differential equations and the use of Monte Carlo methods. We also show that, under the proposed setting, the price of European options can be expressed as the average Black-Scholes option prices. Numerical examples with synthetic and real data illustrate the effectiveness of the pricing and estimation tools.

Diffusing Motion Artifacts for unsupervised correction in brain MRI images

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Keywords: Magnetic resonance imaging; Motion correction

Motion artifacts are a longstanding obstacle in MRI, degrading image quality and leading to diagnostic uncertainty or costly re-scans. Despite the promise of deep learning solutions, most current approaches rely on supervised train- ing, which demands paired motion-free and motion-corrupted images, a type of data that is extremely rare in clinical practice. This scarcity presents a major roadblock for applying these methods broadly, particularly in routine hospital workflows where patient motion is unpredictable and acquisition pa- rameters vary. In this talk, Ill introduce a framework designed specifically to overcome this data bottleneck. Instead of requiring matched image pairs or k-space access, it uses diffusion models to simulate realistic motion arti- facts on clean images, producing synthetic pairs that can then be used to train correction models. This unsupervised pipeline sidesteps the need for controlled acquisition experiments and is compatible with a wide range of clinical scans and hardware setups. Ill present key findings from our eval- uation across datasets and acquisition planes, and discuss why solving the data availability challenge is crucial for making AI-based motion correction a practical reality in MRI.

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Data-Driven Regularization Techniques for Linear Inverse Problems

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Keywords: data-driven regularization; projection methods

We investigate linear inverse problems in scenarios where the forward operator is not explicitly available but is instead represented implicitly through a set of input-output training pairs [1]. Within this data-driven framework, we demonstrate that both projection-based regularization and variational regularization methods can be formulated solely using the available training data, without requiring direct access to the underlying forward operator. We analyze the convergence and stability of the resulting regularized solutions. Furthermore, we show, both analytically and through numerical experiments, that regularization by projection can successfully approximate linear operators such as the Radon transform, effectively enabling the learning of inverse mappings from data alone.

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Stability-Aware Deep Learning Strategies for Image Deblurring under Uncertain Noise

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Keywords: deblur, neural networks, stability, green AI.

The solution of linear inverse problems arising, for example, in signal and image processing is a challenging problem since the ill-conditioning amplifies, in the solution, the noise present in the data. Recently introduced algorithms based on deep learning overwhelm the more traditional model-based approaches in performance, but they typically suffer from instability with respect to data perturbation. In addition, networks do not necessarily take into account the numerical formulation of the underlying imaging problem, when trained end-to-end. In this talk, we propose some strategies to improve stability without losing too much accuracy to deblur images with deep-learning based methods.

First, we suggest a very small neural architecture, which reduces the execution time for training, satisfying a green AI need, and does not extremely amplify noise in the computed image. Second, we introduce a unified framework where a pre-processing step balances the lack of stability of the following, neural network-based, step. Two different pre-processors are presented: the former implements a strong parameter-free denoiser, and the latter is a variational model-based regularized formulation of the latent imaging problem. This framework is also formally characterized by mathematical analysis. Numerical experiments are performed to verify the accuracy and stability of the proposed approaches for image deblurring when unknown or not-quantified noise is present; the results confirm that they improve the network stability with respect to noise.

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Image deblurring under multiplicative noise

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Keywords: Multiplicative noise; Image processing

We consider variational deblurring of images that are corrupted by multiplicative noise. This typically includes minimizing a functional, which is the sum of two further functionals, a data fidelity term and a regularizer. The data fidelity is chosen according to the expected type of noise and is often constructed based on statistical considerations. In a continuous setting, it often takes the form of an integral-based similarity measure between the reconstruction and observation. However, in this infinite-dimensional setting, many of these models have only been studied for the denoising problem. In this talk, we review multiple data fidelity terms used for multiplicative noise removal and introduce a general framework that allows us to extend the theoretical analysis of the respective denoising problems to the ill-posed deblurring problems. As regularizers, we consider total (generalized) variation and related functionals for which some form of Poincaré-Wirtinger inequality holds. We show the existence of minimizers in the variational regularization methods, as well as stability and convergence results. Crucially, we make relatively mild assumptions on the involved blurring operators.

A Bayesian Revolution in the Age of Deep Learning

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Keywords: Bayesian Imaging, Sampling, Learned distributions

Many inverse-problem solvers still chase a single minimiser of a learned objective. However, the single minimizer can be misleading when there are modelling errors, data gaps, or out-of-distribution conditions. Treating modern generative networks as probabilistic priors replaces point estimation with posterior sampling, yielding the entire family of solutions consistent with data and physics. Posterior sampling needs two ingredients: expressive priors that can be learned from data, and scalable algorithms that navigate the resulting high-dimensional, non-convex landscapes. In this talk, we cover recent approaches to these problems. Specifically, we show how the modeling can be addressed by deep neural networks and more classical fields-of-experts models. For both, we show how recent advances in nonsmooth and nonconvex sampling enable the efficient learning of the prior as well as the sampling of the posterior by combining ideas from diffusion models, nonsmooth Langevin sampling, and latent variable models. Finally, we discuss how the adaptation of the model size can be a potential avenue to good generalization when there is little data available. Recasting reconstruction as stochastic exploration, not deterministic optimisation, thus opens a clearer, more reliable path for inverse problems in the deep-learning era: A Bayesian revolution.

Regularized Invertible Neural Networks as Bayesian Point Estimators

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Keywords: Invertible Neural Networks; Bayesian Inverse Problems

Can regularization terms in the training of Invertible Neural Networks (iNNs) lead to known Bayesian point estimators in reconstruction? The use of iNNs for solving inverse problems is desirable due to their inherent stability and interpretability. Recently, optimization strategies for iNNs that approximate either a reconstruction map or the forward operator have been studied from a Bayesian perspective [1]. In the latter case, the influence of the underlying data distribution has been found to be limited.

To address this, we propose regularization terms for training iNNs and show that inverting the resulting network yields approximations of specific Bayesian point estimators such as the MAP depending on the regularizer. We discuss the benefits and limitations of this approach and support our theoretical findings with numerical experiments using iResNets [2].

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Inverse Problem of Parameter Calibration from Diverse Observations for Complex Hydrocarbon Mixtures

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Keywords: Bayesian calibration; Gaussian processes; uncertainty quantification; inverse problems; data-driven modeling

We address the inverse problem of estimating binary interaction parameters (BIPs) in flash calculations for multicomponent hydrocarbon mixtures, a critical step in industrial applications such as flow assurance, reservoir simulation, and chemical process design. Traditional deterministic approaches rely on gradient-based optimization to match experimental data but often neglect uncertainties and unmodeled physics. In contrast, we adopt a Bayesian calibration framework that systematically accounts for observational and modeling errors.

A Gaussian process surrogate approximates the thermodynamic simulator, alleviating the computational cost of repeatedly solving a stiff, nonlinear system. We then use a second Gaussian process to capture unmodeled physical phenomena and measurement noise. Combining these components in a Markov Chain Monte Carlo scheme yields posterior distributions for both model parameters and observational errors. The result is a data-driven, adaptive approach that can update predictions as new observations emerge, enhancing reliability and decision-making in complex industrial processes through richer insight and improved uncertainty quantification compared to single-point deterministic estimates.

Controlled Latent Diffusion Models for 3D Porous Media Reconstruction

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Keywords: First keyword; Second keyword

Inverse problems are pivotal in porous media analysis, demanding the reconstruction of subsurface microstructures from limited imaging datacritical for applications such as energy exploration, hydrocarbon recovery, and underground storage. While traditional approaches often rely on statistical or physics-based models, recent breakthroughs in deep generative methodsparticularly diffusion-based approachespresent new avenues for enhanced accuracy and computational efficiency.

In this talk, I will introduce a novel framework for reconstructing 3D porous microstructures using controlled latent diffusion models, developed in collaboration with ExxonMobil. By integrating the EDM diffusion framework with a tailored variational autoencoder (VAE), we achieve high-resolution digital rock reconstructions while preserving computational feasibility. Furthermore, we propose controlled unconditional sampling, an approach that improves model reliability through the incorporation of physical constraints such as porosity and two-point correlation functions.

Our results demonstrate that generative models can act as powerful, data-driven solvers for inverse problems, addressing challenges where traditional physics-based models falter due to data scarcity or high computational costs. This methodology is broadly applicable across geophysics, medical imaging, and materials scienceany field where reconstructing complex microstructures from indirect observations is essential.

Inverse Methods in Multiscale Modeling of Heterogeneous Porous Media: Key to Advancing Energy, Agriculture, and Environmental Sustainability

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Keywords: Multiscale modeling; Heterogeneous porous media; Transport in natural systems.

Estimating the effective properties of heterogeneous porous media remains one of the most intricate challenges in modeling systems involving fluid flow, reactive transport, and thermal exchange. These porous structures are inherently variable and anisotropic, often requiring sophisticated multiscale modeling frameworks to bridge information from the pore to the field scale. However, the real leap in predictive capability lies in the integration of inverse methods, which allow us to infer critical but non-directly measurable parameters from observed data.

From petroleum engineering to environmental sciences and agriculture, many physical parameters such as permeability distributions, capillary pressures, and saturation functions cannot be measured directly at all relevant scales. Here, inverse modeling techniques become indispensable. By calibrating numerical models with field or laboratory data (such as pressure transients, saturation profiles, or production curves), inverse methods enable the reconstruction of spatially variable properties that govern flow and transport.

Complex geological features (fractures, vugs, and facies changes) significantly impact fluid dynamics in subsurface energy applications, such as enhanced oil recovery or carbon storage. Traditional forward simulations fall short in accounting for this complexity. Inverse methods allow these multiscale models to be tuned against real-world data, enhancing their fidelity and interpretability. For example, in pre-salt reservoirs, pore-scale simulations using tools like Lattice Boltzmann or Pore Network Models provide detailed geometrical and physical insights, but their usefulness at the reservoir scale depends on upscaling and parameter estimationtasks that inverse methods help solve rigorously.

Upscaling, by nature, leads to a loss of fine-scale information. Inverse modeling mitigates this loss by adjusting large-scale models to maintain consistency with observational data. This results in more robust predictions and tighter uncertainty bounds, particularly important in high-stakes decision-making environments like field development planning or CO_2 injection projects. Moreover, inverse calibration plays a vital role in the history-matching of field production, interpretation of pressure tests, and understanding of inter-well connectivity.

Beyond the energy sector, inverse methods are equally transformative in fields like agriculture and hydrology. In soil science, for example, inverse approaches help deduce the spatial variability of retention and conductivity curves from irrigation experiments, enhancing the design of water-efficient agricultural systems. In hydrogeology, inverse models are essential for reconstructing subsurface hydraulic properties from sparse observations, enabling better management of aquifers and contaminant transport assessments.

In carbon capture and storage (CCS), where predicting CO_2 plume migration over decades is critical for both safety and regulatory compliance, inverse modeling based on seismic, pressure, and tracer monitoring data provides a powerful way to validate and update multiscale simulations in real-time.

Integrating advanced inverse techniques with emerging computational toolssuch as Bayesian inference, machine learning, and hybrid physics-informed modelshas dramatically expanded their potential. These approaches not only automate and accelerate the calibration process but also enable probabilistic characterizations of uncertainty, which are crucial in systems where data are sparse or noisy.

In conclusion, the synergy between multiscale modeling and inverse methods forms a cornerstone of modern porous media research. By allowing us to reconstruct hidden parameters from observable behavior, inverse modeling enhances the realism, reliability, and applicability of simulations in fields ranging from energy and agriculture to hydrology and climate mitigation. This methodological integration is key to reducing uncertainty, improving decision-making, and advancing sustainable technologies.

From X-ray Visibilities to Electron Maps: An Inverse Approach with STIX Data

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Keywords: Spectral inversion; Imaging-spectroscopy; Solar flares

Solar flares are intense explosions occurring on the solar surface, which are produced by reconnection of magnetic field lines. During these events, accelerated electrons interact with heavy particles, and consequently X-ray photons are emitted by bremsstrahlung.

The Spectrometer/Telescope for Imaging X-rays (STIX, [1, 2]) on-board the ESA Solar Orbiter mission, modulates the X-ray radiation emitted during solar flares by means of a bi-grid system. STIX observations represent spatial Fourier components of the incoming photon flux, named *visibilities*. Obtaining information on flare-accelerated electrons is one of the fundamental scientific objective of solar X-ray imaging spectroscopy.

In this talk, we present a method for reconstructing maps of the electron flux distribution directly from STIX visibilities. We obtain images whose pixel values are proportional to the number of accelerated electrons by addressing two nested inverse problems: first, the reconstruction of electron visibilities from X-ray visibilities, and second, the inversion of these into spatially-resolved electron maps [3]. The resulting maps enable the extraction of important physical information on the evolution of solar flares, such as the flare acceleration effectiveness [4].

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Radio-interferometric data processing in the ngVLA era

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Keywords: Radio-interferometry; HPC computing, Imaging

The correlated signals of two antennas in a radio interferometer array approximates the Fourier transform of the on-sky emission at a spatial frequency determined by the baseline separating the antennas projected to the sky plane. Due to the limited number of antennas, additional noise, and a variety of calibration effects, the data processing is a challenging ill-posed inverse problem. They have been traditionally solved by monolithic, and mainly manual calibration and imaging applications. However, the next-generation of radio interferometers like ngVLA will operate at much higher data rates and sensitivities challenging current interferometric data processing pipelines. Scalability to the Petaflops regime per second may be necessary. These developments are supported by the ongoing push towards science ready data products (SRDP), calling for high performance, and automatized batch processing of huge data sets. The National Radio Astronomy Observatory is actively preparing for this era of big-data processing and is laying out the algorithmic and pipeline-heuristical foundations already now. In this talk, I will review recent approaches to data processing to scale algorithms towards the next generation of flagship instruments. These include a novel software infrastructure envisioned to replace CASA, especially developed for scalable deployment on GPUs. Calibration, and particularly RFI flagging could supersede the deconvolution as the limiting factor for the images dynamic range and needs to be performed iteratively with the imaging deploying the same GPU architecture. Imaging-wise, while imaging with traditional matching pursuit techniques was the to-go method for decades, in view of increased sensitivity and dynamic range, the assumption inherent to CLEAN saturate the performance unacceptably. The development of efficient, scalable and fast wide-band and wide-field multiscale deconvolution algorithms is paramount. Much anticipated convergence acceleration is achieved by clustering components informed by Bayesian heuristics, applying concepts and ideas of convex optimization to the CLEAN framework, and the utilization of AI for efficient compressing among the spatial and spectral domain. Finally, I will discuss pathways to automatization of pipelines and some computing logistics to streamline the downstream analysis.

Deep Learning for Solar Active Region Classification

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Keywords: 65Z05; 68T07; 85-08

Solar active regions can deeply impact the SunEarth space environment, often triggering intense space weather events such as solar flares and coronal mass ejections. In this talk, we discuss how deep learning methods can be employed to classify solar active region images for flare forecasting purposes. To this aim we examine cutting-edge image classification architectures from convolutional neural networks to vision transformers and incorporate modern models training strategies like on-the-fly data augmentation and transfer learning [1]. We address both the task of classifying active region from image cutouts and the detection and classification of active regions from full-disk images. This research is conducted as part of the EU-funded Active Region Classification and Flare Forecasting (ARCAFF) project.

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Image reconstruction with (learned) regularisation parameter maps

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Keywords: Image reconstruction, adaptive regularisation, statistical approaches, noise whiteness, deep learning

In this talk I will present some recent work on the estimation of adaptive (i.e. pixel-dependent) regularisation map in image reconstruction. At first, I will present some of the approaches in [1] which rely on maximum-likelihood-type estimation of local strength, smoothness and anisotropy parameters for exemplar Total Variation (TV) regularisation frameworks. Then, I will discuss some more recent work [3, 2] where a reference-free functional modelling noise whiteness is used for unsupervised estimation of adaptive maps. To conclude, I will discuss how deep-learning approaches can be used to estimate parameter maps either by means of shallow architectures [4] and patch-based approaches or, upon deep reparametrisation, using the unrolling of a FISTA-type algorithm in the case of synthesis-based approaches [5].

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TILT: Topological Interface Recovery in Limited-Angle Tomography

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Keywords: Limited-angle; Tomography; Topology

I present a reconstruction method for the severely ill-posed inverse problem of limited-angle tomography. Depending on the available measurement, angles specify a subset of the wavefront set of the unknown target, while some oriented singularities remain invisible in the data. Topological Interface recovery for Limited-angle Tomography (TILT) is based on lifting the visible part of the wavefront set under a universal covering map. In the space provided, it is possible to connect the appropriate pieces of the lifted wavefront set correctly using complex wavelets, a dedicated metric, and persistent homology. The result is not only a suggested invisible boundary but also a computational representation for all interfaces in the target. The talk is based on the article [1].

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A real-time method for ventilation and bloo pulsatility separation in functional images of the chest

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Keywords: First keyword; Second keyword

Mapping the ventilation and the regional blood pulsatility behavior of the lung is essential for treating patients with respiratory failure, especially those under mechanical ventilation. The separation of ventilation and blood pulsatility in Electrical Impedance Tomography images of the thorax is the focus of this paper, with potential applications in medical imaging and other tomographic imaging modalities. The method has two stages. In the first stage, the algorithm is trained to identify the dynamic models of ventilation and pulsatility cycles separately. The second stage uses the adjusted models to separate new incoming images in real-time. During the training stage, two average cycles are estimated - one for ventilation and the other for blood pulsatility. These average cycles are then used to adjust evolution models for real-time processing in the second stage. The proposed method was evaluated with experimental data in swines under mechanical ventilation.

Dual-grid parameter choice method with application to deblurring

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Keywords: parameter choice; regularization; deblurring

Finding an optimal regularization parameter to balance between the data fidelity and a regularization term is still generally an open question. We present a new parameter choice method based on the use of two slightly different computational models for the same inverse problem. Small parameter values should give two very different reconstructions as noise is amplified. Large parameter values lead to identical but trivial solutions. Matching image similarity of the reconstructions with a pre-defined threshold value we can choose an optimal parameter between the two extremes. The effectiveness of the method is demonstrated with the image deblurring task using simulated and real measured data and two different regularizers.

Doubling Inequality for Nanoplate Equations

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Keywords: Higher-Order Elliptic Equations; Nanoplates, Strong Unique Continuation,

We analyze the property of Strong Unique Continuation for a sixth order elliptic operator arising in the framework of the strain gradient linear elasticity theory for nanoplates in flexural deformation in the case of isotropic materials, [3]. We obtain such a property of Strong Unique Continuation in the quantitative form of doubling inequality and three spheres inequality by a Carleman estimates approach, [1], [2].

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Size estimates for nanoplates

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Keywords: Size estimates; Elastic nanoplates; Unique continuation .

We consider the problem of determining, within an elastic isotropic nanoplate in bending, the possible presence of an inclusion made of different elastic material. Under suitable a priori assumptions on the unknown inclusion, we provide quantitative upper and lower estimates for the area of the unknown defect in terms of the works exerted by the boundary data when the inclusion is present and when it is absent. This presentation is based on the results contained in [1, 2] .

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Reconstructing Loads in Nanoplates from Dynamic Data

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Keywords: Load reconstruction, nanoplates, strain-gradient elasticity, linear dynamics

In this work, we address an inverse source problem for nanoplates modeled within the strain gradient linear elasticity theory, under the Kirchhoff–Love kinematic assumptions. It was recently proven that the knowledge of the transverse displacement on an open subset of the mid-plane, over any time interval, uniquely determines the spatial components of a separable transverse load of the form

$$\sum_{m=1}^{M} g_m(t) f_m(x,y),$$

where the temporal functions $\{g_m(t)\}_{m=1}^{M}$ are known and linearly independent. Building on this theoretical result, we propose a reconstruction algorithm for identifying the spatial load components $\{f_m(x,y)\}_{m=1}^{M}$. The numerical implementation relies on a finite element spatial discretization and is carried out for a uniformly thick rectangular nanoplate clamped along its boundary. We perform a detailed sensitivity analysis with respect to the key parameters influencing the identification process. The adoption of a regularization strategy based on the singular value decomposition proves to be crucial for achieving both accuracy and stability in the reconstruction.

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Identification of the mass density in a nanoplate from finite eigenvalue data

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Keywords: Nanoplates; Inverse eigenvalue problems; Finite data; Mass identification

We present a reconstruction method for the determination of a mass density perturbation in a nanoplate resonator. The method uses as data the first lower resonance frequency shifts produced by the added mass on a uniform rectangular nanoplate subjected to transverse vibration and simply supported at the boundary. The support of the mass variation is assumed to be within a quarter of the nanoplate domain and a simplified theory of the strain gradient in linear elasticity is used to account for size effects. The reconstruction is based on a sequence of linearizations of the inverse eigenvalue problem in a neighborhood of the initial nanoplate. The limitations and potential of the numerical reconstruction method for various types of mass perturbations are illustrated and discussed.

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Minkowski metric is rigid in the Lorentzian Calderón problem

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Keywords: Calderón problem; Lorentzian geometry

We study the Lorentzian Calderón problem, where the objective is to determine a Lorentzian metric up to a boundary fixing diffeomorphism from boundary measurements given by the hyperbolic Dirichlet-to-Neumann map. We prove that if a globally hyperbolic metric agrees with the Minkowski metric outside a compact set and has the same hyperbolic Dirichlet-to-Neumann map as the Minkowski metric, then it must be the Minkowski metric up to diffeomorphism. In fact, we prove the same result with a much smaller amount of measurements, thus solving a formally determined inverse problem. To prove these results we introduce a new method for hyperbolic inverse problems. The method is based on distorted plane wave solutions and on a combination of geometric, topological and unique continuation arguments.

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Geometry of gas giants and inverse problems

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Keywords: First keyword; Second keyword

In this talk, I will introduce a Riemannian geometric model for wave propagation in gas giant planets. Terrestrial planets and gas giants have one key difference: in gas, density goes to zero at the surface, and seismic waves come to a full stop. We model the sound speed in a planet by a Riemannian metric. Starting from a polytropic model for the planet, we derive that the vanishing of the density amounts to a specific conformal-type singularity in the Riemannian metric. We will highlight the key differences between the arising geometry and its more studied relatives. We finish with an overview of inverse problems results in our new geometry. The talk is based on joint work with Maarten de Hoop (Rice University), Joonas Ilmavirta (University of Jyväskylä), and Rafe Mazzeo (Stanford University).

Reconstruction of the observable Universe from the Sachs-Wolfe effects

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The integrated Sachs-Wolfe (ISW) effect is a property of the Cosmic Microwave Background (CMB), in which photons from the CMB are gravitationally redshifted, causing the anisotropies in CMB. An intriguing question is what information of the gravitational perturbations can be inferred from the ISW effect. In this talk, we explore the possibility of a tomography approach, similar to the X-ray CT in medical imaging. In particular, we consider the X-ray transform for null-geodesics in Lorentzian geometry. With the help of the PDE model for CMB, we prove the stable inversion of the transform and address the partial data problem with observations only near the Earth. Also, we discuss recent advances and some challenges in the numerical simulation as well as related geometric inverse problems.

Double fibration transforms with conjugate points

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Keywords: double fibration transform; Bolker condition; conjugate point

In this talk we discuss the structure of normal operators of double fibration transforms with conjugate points. Examples of double fibration transforms include Radon transforms, d-plane transforms on the Euclidean space, geodesic X-ray transforms, light-ray transforms, and ray transforms defined by null bicharacteristics associated with real principal type operators. We show that, under certain stable condi- tions on the distribution of conjugate points, the normal operator splits into an elliptic pseudodifferential operator and several Fourier integral operators, depending on the degree of the conjugate points.

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Seeing nonlinearities

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Keywords: nonlinear PDE; nonlinear optics, weakly nonlinear geometric oprics, DC Kerr effect

I will present results on recovery of nonlinearities in nonlinear hyperbolic PDEs. They include models of nonlinear acoustics and nonlinear optics, among the rest. The main idea is to prove nonlinear effects observed in physics, derived from the properties of exact solutions with a small parameter proportional to the wavelength. The nonlinearity affects the leading term in a very essential way. Existence of such solutions is quite technical since they are not "small" but they are the physically relevant ones. Once we construct the proper ansatz and prove existence of exact solutions with that asymptotic behavior, the solution of the inverse problem is immediate.

In particular, I will show how one can derive the DC Kerr effect in nonlinear optics from the nonlinear Maxwell system: a strong constant electric field changes the polarization of a light beam by nonlinear interaction. This effect is the theoretical basis of the Kerr cells used to measure speed of light in lab conditions. The nonlinear effect can be literately seen.

The talk is based on works in collaboration with Nikolas Eptaminitakis and Antônio Sá Barreto.

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Lipschitz stability for the determination of coefficients and elastic inclusions

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Keywords: Lipschitz stability; Calderón problem; Schrödinger equation; Elastic inclusions

Generally, inverse boundary value problems for PDE are severely ill posed, which poses major difficulties in the numerical reconstructions. Considering the unknowns in certain finite-dimensional spaces provides an improvement in terms of stability, which becomes indeed of Lipschitz type. In this context, I will first focus on both the Calderón problem and the inverse problem for the Schrödinger operator discussing some Lipschitz stability estimates for conductivities and potentials depending on a finite number of parameters by means of infinitely many local boundary measurements. I will then introduce a recent Lipschitz stability result for the determination of a polyhedral inclusion in an elastic body by the knowledge of the local Dirichlet to Neumann map. This talk is based on a series of joint works with G. Alessandrini, A. Aspri, E. Beretta, M. V. de Hoop, S. Foschiatti, E. Francini, R. Gaburro, A. Morassi, E. Rosset, A. Rüland and S. Vessella.

A Sensitivity-Based Algorithm Approach in Reconstructing Images in Electrical Impedance Tomography

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Keywords: Electrical impedance tomography; gateaux derivative; image reconstruction; numerical algorithm; sensitivity

Electrical impedance tomography (EIT) is a medical imaging technique used to reconstruct images inside the domain of interest. EIT collects data on the boundary of the domain to infer the conductivity distribution inside the domain. The conductivity distribution will then be used to produce a tomographic image of the inside of the domain. This paper aims to recover geometric properties of a spherical perturbation in the conductivity inside a domain using sensitivity values of the electric potential on the boundary of the domain. The continuum model for EIT is first considered, as it holds more boundary information compared to other models of EIT. A change on the conductivity inside the domain is applied, and the impact on the electric potential is studied. The inverse EIT problem is then solved by formulating relations between the sensitivity values on the boundary and the geometric properties of the spherical perturbation: the radius and the projection onto the boundary and depth of its center. A reconstruction method using these relations is proposed and the method is examined by performing numerical simulations on different domains to model the head and the thorax. Lastly, the proposed method is applied to the complete electrode model of the EIT problem to analyze the performance of the method when the boundary data is limited on the electrodes.

Stability estimates of a stochastic inverse source problem for a class of elliptic multipliers

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Keywords: Inverse problems, stochastic sources, stability estimates, numerical reconstruction

Abstract

We consider an inverse source problem for a class of differential operators of the form

$$\lambda I + \sum_k c_k (-\Delta)^{s_k},$$

where $\lambda \in \mathbb{R} \setminus \{0\}$, $s_k \in \mathbb{N}_0$, and $c_k > 0$. The source term is modeled as a stochastic process, accounting for uncertainty in the underlying data. Our goal is to reconstruct the statistical properties of the source—specifically, its mean and variance—based on noisy measurements of the solution. We derive explicit reconstruction formulas and prove stability estimates that guarantee robustness with respect to data perturbations. A numerical experiment in one spatial dimension illustrates the practical applicability of the proposed method.

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Sensitivity analysis of the complete electrode model for electrical impedance tomography

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Keywords: electrical impedance tomography, complete electrode model, sensitivity analysis, conductivity, contact impedance

Electrical impedance tomography (EIT) is an imaging technique that reconstructs the conductivity distribution in the interior of an object using electrical measurements from the electrodes that are attached around the boundary. The Complete Electrode Model (CEM) accurately incorporates the electrode size, shape, and effective contact impedance into the forward problem for EIT. In this work, the Gâteaux derivative is introduced as a tool for the sensitivity analysis and the Gâteaux differentiability of the electric potential with respect to the conductivity and to the contact impedance of the electrodes is proved. The derivative is then expressed as the unique solution to a variational problem and the discretization is performed with Finite Elements of type P1. Numerical simulations for different 2D and 3D configurations are presented. This study illustrates the impact of the presence of perturbations in the parameters of CEM on EIT measurements.

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Source and Boundary Values Control each other

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Keywords: Python Language; Differential Equations; Boundary value problem; Inverse Problem

Assuming a physical variable that satisfies a certain boundary value problem.For some purpose, we want to control the over specified boundary values through appropriate values to the source. On the other hand, for a purpose, we want to determine an unknown source based on boundary values that are known by measurement, or we want them specifically. It is clear that this is an inverse problem, and in order to obtain a single solution to this problem, we will restrict ourselves to a specific space to which this source belongs. In this research, we will explain the theoretical method to solve this problem and present two models for these boundary value problems. For each model, we give examples and attach Python codes for these examples.

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Utilizing the Monte Carlo method for light transport to estimate absorption and scattering in optical tomography

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Keywords: Image reconstruction techniques; Monte Carlo; Optical tomography

In optical tomography, optical properties of an imaged target are estimated from boundary measurements of visible or near-infrared light. The method has potential applications, for example, in functional brain studies, breast cancer imaging, and small animal studies. A numerical solution of the inverse problem in optical tomography requires modeling of light transport. Generally, the diffusion approximation of the radiative transport equation is used. However, in the so-called transport regime, when the target size is less than few scattering lengths, the diffusion approximation is not valid, and light transport needs to be modeled as radiative transfer.

In this work, the Monte Carlo method for light transport is used to numerically approximate the solution of the radiative transport equation. The Monte Carlo method for light transport is a stochastic method where paths of photons (or photon packets) are simulated as they undergo absorption and scattering events in a scattering medium. In the inverse problem of optical tomography, the spatial distributions of absorption and scattering are estimated. The corresponding minimization problem is solved using a stochastic Gauss-Newton method. To calculate the derivatives for scattering, a perturbation approximation for the Monte Carlo method is utilized. The proposed method is evaluated with numerical simulations using an adaptive approach, where the number of photon packets in the Monte Carlo simulations is selected automatically during the iterations, and with different fixed numbers of photon packets.

Simultaneous reconstruction of initial pressure and speed of sound distributions in photoacoustic tomography

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Keywords: Photoacoustic Tomography; Ultrasound Imaging; Image Reconstruction

Photoacoustic tomography (PAT) is a hybrid imaging modality that combines the unique optical contrast with the high spatial resolution of ultrasound. In PAT measurements, propagating ultrasound waves are triggered in a target by using an external irradiation of near-infrared light. Solution of the PAT inverse problem, i.e. reconstructing the initial pressure from measured ultrasound waves, relies on the knowledge of the speed of sound of the target. Since the speed of sound is often unknown in practice, it would be convenient to reconstruct it simultaneously with the initial pressure. However, reconstructing the initial pressure and speed of sound simultaneously is highly difficult to do in practice due to the ill-posed nature of the problem.

In this work, an approach where multiple initial pressure distributions in the imaged target are generated, and the produced dataset is used in the solution of the inverse problem, is proposed. Then, the speed of sound is reconstructed simultaneously with different initial pressures. The approach is evaluated with numerical simulations, and compared to a reference approach where a single initial pressure generated in the target is utilised to produce the measurement dataset.

Effects of inertia in implicit iterative methods: experiments on a nonlinear parameter identification

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Keywords: Nonlinear problems; Cyclic Methods; Stochastic Methods; Inertial Methods; Implicit Methods

Several inverse problems involve operators with ranges that can be decomposed into multiple components, for which well-established methods, such as cyclic and stochastic Kaczmarz approaches like explored in [1] and [2] respectively, are commonly used. Recent studies as [3] and [4] have examined the effect of inertia—an extrapolation parameter applied at each step - on various iterative methods. In this work, we present numerical experiments on a nonlinear PDE parameter identification problem, investigating how different inertia parameters affect the performance of implicit methods that utilize full batch, cyclic, and stochastic approaches at each step. Our results show that higher inertia leads to slight improvements across all scenarios, particularly for cyclic and stochastic methods.

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On Gradient-Type Projection Methods for Nonlinear Ill-Posed Problems

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Keywords: Ill-posed problems; Nonlinear equations; Projection methods; Tangential cone condition

We analyze a family of successive projection methods for solving nonlinear ill-posed problems with convex constraints in Hilbert spaces [1]. The methods consist of a nonlinear Landweber iteration for the reduced problem in the intersection of the null spaces of the active constraints and a projection step. This formulation is particularly suitable for parallel computing. We prove convergence for both exact and noisy data and derive convergence rates using conditional stability estimates. Finally, we present numerical examples with parameter identification problems in elliptic PDEs which confirm the robustness and efficiency of the proposed method.

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An a-posteriori criteria for choosing Lagrange multipliers in nonstationary iterated Tikhonov method

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Keywords: Ill-posed problems, Linear operators, Iterated Tikhonov method, Nonstationary methods.

We investigate nonstationary iterated Tikhonov-type method for obtaining stable approximate solutions to ill-posed operator equations modeled by linear operators acting between Hilbert spaces [**?**]. Geometrical properties of the operator modeling the problem are used to derive a new strategy for choosing the sequence of regularization parameters (Lagrange multipliers) for the NIT iteration. Convergence analysis for this method is provided. Numerical experiments are presented for two distinct applications, namely (I) a two-dimensional elliptic parameter identification problem (inverse potential problem); and (II) an image-deblurring problem. The results obtained validate the efficiency of our method compared with standard implementations of the NIT method (where a geometrical choice is typically used for the sequence of Lagrange multipliers).

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Data-Proximal Neural Networks for Limited-View CT

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Keywords: Limited-angle CT; Data-Proximal Networks, Reconstruction

Image reconstruction in limited-angle computed tomography (CT) is a challenging inverse problem due to its ill-posedness and underdetermined nature. While deep learning methods have shown impressive empirical performance, they often lack theoretical guarantees—particularly regarding data consistency, which is crucial in safety-critical applications such as medical imaging. In this poster, we present a data-proximal network architecture recently introduced in [**?**, **?**], which integrates a mathematically rigorous regularization framework into the learning process. This architecture enforces proximity to the measured data and thus ensures consistent reconstructions. We demonstrate the effectiveness of our approach on limited-angle CT and compare its performance with a standard residual network and a null space network, highlighting its advantages in both reconstruction quality and data fidelity.

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Inexact Newton regularizations with uniformly convex stability terms: A unified convergence analysis

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Keywords: Nonlinear inverse problems; Inexact Newton regularization; Convex optimization; Bregman distances; Electrical impedance tomography.

In this work, we present a unified convergence analysis of inexact Newton regularizations with general uniformly convex penalty terms for nonlinear ill-posed problems in Banach spaces. These schemes consist of an outer (Newton) iteration and an inner iteration, which provides the update of the current outer iterate. To this end, the nonlinear problem is linearized about the current iterate and the resulting linear system is inexactly solved by an inner regularization method. In our analysis, we only rely on generic assumptions of the inner methods, and we show that a variety of regularization techniques satisfies these assumptions. For instance, gradient-type and iterated-Tikhonov methods are covered. Numerical experiments based on the inverse problem of electrical impedance tomography illustrate the impact of different uniformly convex penalty terms.

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Do we need the Adjoint to Estimate Operator Norms and Differences?

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Keywords: Norm Estimation; Adjointfree; Stochastic Methods

In this poster, we tackle two related inverse-problem challenges:

1. estimating the norm of a linear operator A with no access to its adjoint, and
2. measuring the distance $\|A - V\|$ when a mismatched surrogate adjoint V^* stands in for the true adjoint A^*.

We present a unified semi-stochastic framework that uses only oracle evaluations of A and V^*, requires no explicit matrix representations, and maintains minimal memory overhead. By carefully selecting step-size rules, our iterative scheme converges almost surely to the exact operator norm or norm difference. Through a suite of numerical experiments, we demonstrate that our method is both computationally efficient and robust across high-dimensional settings where adjoint consistency is not guaranteed—making it a versatile tool for primal–dual optimization and broader inverse-problem applications.

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Sequential Optimal Experimental Design for Single-Pixel Imaging using Reinforcement Learning

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Keywords: optimal experimental design; Bayesian inverse problems; single-pixel imaging

Single-pixel imaging (SPI) is cost-efficient, highly compressive measurement technique with potential advantages in various imaging fields [**?**]. We develop a framework for the adaptive design of a family of optimal linear binary-entry SPI sensing operators. Our proposal describes a way to find optimal patterns by maximizing a sequential utility functional. By quantifying the certainty a pattern would add to the distribution of possible images conditioned on the measurements acquired, we address technical and numerical challenges for large-scale single-pixel imaging.

Our work revolves around the *single-pixel imaging* inverse problem of recovering an image $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^M$ from measurements $\mathbf{y} \in \mathcal{Y} \subseteq \mathbb{R}^N$ obtained via

$$\mathbf{y} = \mathcal{N}oi(\mathbf{H}\mathbf{x}) \tag{1}$$

where $\mathcal{N}oi$ defines the statistical noise process this acquisition involves and $\mathbf{H} \in \mathcal{H} = \{-\mathbf{1}, \mathbf{1}\}^{N \times M}$ is a binary-entry linear matrix.

The likelihood distribution under the design $\mathbf{H}$, $P_{\mathbf{H}}(\mathbf{y}|\mathbf{x})$, provides a full description of the possible measurements $\mathbf{y}$ given the unknown image $\mathbf{x}$ under the linear relation $\mathbf{H}$. In the Bayesian framework, the solution of this inverse problem can be fully expressed via the posterior distribution $P_{\mathbf{H}}(\mathbf{x}|\mathbf{y})$:

$$P_{\mathbf{H}}(\mathbf{x}|\mathbf{y}) = \frac{P_{\mathbf{H}}(\mathbf{y}|\mathbf{x})P(\mathbf{x})}{P_{\mathbf{H}}(\mathbf{y})} \tag{2}$$

The optimality criteria to choose a static, *a priori* (i.e., before measurement time) design $\mathbf{H}$, can be generally described as the optimal experimental design problem of finding $\mathbf{H}^*$ s.t.

$$\mathbf{H}^* = \mathrm{argmax}_{\mathbf{H} \in \mathcal{H}} \mathbb{E}_{P_{\mathbf{H}}(\mathbf{y})} \left[\mathcal{U}(\mathbf{H}, \mathbf{y})\right] \tag{3}$$

where $\mathcal{U} : \mathcal{H} \times \mathbb{R}^N \to \mathbb{R}_{\geq 0}$ is the utility functional, a goodness quantification of the design choice $\mathbf{H}$. The conditional expected value $\mathbb{E}_{\mathbf{y}|\mathbf{H}}[\cdot]$ produces an average estimate over the possible measurements $\mathbf{y}$ that the design $\mathbf{H}$ would lead to.

In sequential optimal experimental design, $\mathbf{H}^*$ is chosen *a posteriori* (i.e., during measurement time) and adapted in batches: Its rows $\mathbf{h}_k^*$, with $0 < k \leq N$, are chosen to maximise the utility $\mathcal{U}_{k,b} : \mathcal{D}_{k-b} \times \mathcal{H}_b \to \mathbb{R}$.

$$\{\mathbf{h}_j^*\}_{j=k-b}^k = \mathrm{argmax}_{\{\mathbf{h}_j\}_{j=k-b}^k \in \mathcal{H}_b} \mathbb{E}_{\mathbf{y}|\{\mathbf{h}_j^*\}_{j=k-b}^k} \left[\mathcal{U}(\mathbf{D}_{k-b}, \{\mathbf{h}_j\}_{j=k-b}^k)\right] \tag{4}$$

We note in this case the usage of a shared utility functional. Now, this batch-wise functional maps from the history $D_{k-b} \in \mathcal{D}_k = \mathcal{H}_{k-b} \times \mathbb{R}^{k-b}$, where $\mathcal{H}_{k-b} = \{-\mathbf{1}, \mathbf{1}\}^{(k-b) \times M}$ is the space of possible past batch of patterns.

For the presented problem, our poster addresses the following:

- A search for a consistent Bayesian sequential optimal experimental design (BsOED) [**?**] of single-pixel imaging binary patterns.
- Construction of SPI-specific strategies for explicit, density-based, and implicit, sample-based priors, and the conditional posterior updates [**?**].
- Reinforcement learning-based methodologies to remap and solve the BsOED utility maximization problem as a partially observable Markov decision process (POMDP) [**?**].
- An exploratory numerical analysis of our proposals to solve the BsOED-POMDP remapping.

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Fine mapping properties of the Radon transform near the boundary of the closed unit ball

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Keywords: Radon Transform; Double b-Fibration; Polyhomogeneous, Conormal Function Classes; Pushforward Theorem; Mellin Functional;

We discuss fine mapping properties of a restricted Radon transform R on conormal classes of functions on the closed unit ball centered at the origin of $\mathbb{R}^n$. We construct a desingularization of the point-hyperplane relation between the closed unit ball $\overline{\Omega}$ and hyperplanes of $\mathbb{R}^n$ using the tangent bundle over the closed cylinder $\overline{Z} = [-1, 1] \times S^{n-1}$ and show it is a double b-fibration over $\overline{\Omega}$ and $\overline{Z}$. We express R and the back-projection R^* with pullbacks and pushforwards generated by the b-fibrations and conormal multipliers to formulate estimates on the mapping properties of R and R^* on polyhomogeneous, conormal classes on $\overline{\Omega}$ and $\overline{Z}$ using a naive application of the Pushforward and Pullback theorems of R. B. Melrose [1]. We then show those estimates are not sharp and apply Melrose's original technique of Mellin functionals to construct sharper ones. We prove new boundary determination and mapping properties for R and the normal operator R^*R on the closed unit ball with key boundary regularity distinctions in even and odd dimension n. These results are in agreement with [2] wherein the Radon transform is a reparameterization of an X-ray transform on the Euclidean closed unit disc in dimension $n = 2$.

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Singularly Weighted Tensor Tomography on the Disk

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Keywords: tensor tomography, X-ray transform, singular weights

We study the problem of tensor tomography on the unit disk equipped with the weight $d(x) = (1 - |x|^2)^\gamma$ where $\gamma \in (-1, 1)$. This problem is solved by inverting the weighted X-ray transform Id^γ. Over tensors, Id^γ is not injective, and from a new Helmholtz decomposition, the identification of gauge for Id^γ is possible. The range of Id^γ is characterized and a fast reconstruction procedure is described.

Some Non-Constant Curvature Disk with Blow-Down Structure

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Keywords: transport twistor space, blow-down structure, β-map

We consider the Riemannian manifold $M = \{(r, \theta) : 0 \leq r \leq 1, 0 \leq \theta \leq 2\pi\}$ with the metric $g = (1 + \kappa r^2)^2 dr^2 + r^2 d\theta^2$. We reconstruct the β-map on the unit sphere bundle[2]. Then, we explore the circle action on the bundle space to extend the β-map to the transport twistor space. Finally, we show that the β-map exhibits a blow-down structure[1].

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Adaptive Spectral Inversion for Eigenfunction Selection in Inverse Source Problems

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Keywords: Inverse problems; Regularisation; Adaptive Eigenspace Inversion; Adaptive Spectral Inversion; Eigenfunction sensitivity ranking

Abstract. Inverse source problems are inherently ill-posed, necessitating the use of regularisation techniques to ensure stable and accurate reconstructions. Adaptive Eigenspace Inversion (AEI) is a spectral regularisation method that reconstructs the source by projecting the problem onto a truncated eigenfunction basis. A key challenge in AEI lies in determining the optimal number of retained eigenfunctions, denoted by M, as no well-established criterion exists to ensure an optimal balance between accuracy and stability. To address this issue, Adaptive Spectral Inversion (ASI) has been proposed as a strategy to rank eigenfunctions based on their sensitivities, thereby refining the selection process. In this study, we evaluate the effectiveness of ASI in improving source reconstruction by systematically analysing its impact on reconstruction accuracy and computational efficiency. Our findings provide a quantitative assessment of ASI's performance, demonstrating its potential to enhance the reliability of inverse source problem solutions.

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Deterministic and Stochastic Optimisation Framework using the Core Imaging Library and Synergistic Image Reconstruction Framework for CT and PET Reconstruction

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Keywords: Tomography; Open-source Software; Optimisation; Python

In recent years, a multitude of complex tomography challenges have emerged, requiring collaborative efforts across mathematics, algorithm design, numerical software development and imaging practitioners. To support this collaboration, we have built the Core Imaging Library (CIL) [**?**, **?**], providing a comprehensive toolkit for solving inverse problems in mathematical and computational imaging. This talk explores some of CIL's successful applications within tomography and general imaging tasks, emphasizing its modular optimization framework and introducing a new stochastic optimization approach.

Firstly, we illustrate our work on hyperspectral neutron tomography [**?**], where CIL is used to resolve materials spatially and spectrally based on Bragg edges in energy-resolved neutron data.

Secondly, we demonstrate a directional total variation reconstruction method in CIL, which won a prize at the Helsinki Tomography Challenge 2022 for limited-angle X-ray CT reconstruction [**?**].

Finally, we present recent advancements in CIL, introducing a framework for stochastic algorithms which have the potential to increase speed and efficiency with the increasingly large datasets encountered with modern imaging methods. The CIL optimisation framework can now integrate stochastic gradient estimators into base algorithms like gradient descent and iterative soft thresholding, so we can switch between stochastic and non-stochastic algorithms; including stochastic gradient descent (SGD), stochastic average gradient (SAG), and stochastic variance reduced gradient (SVRG). The plug-and-play nature of the software library enables easy comparison between different stochastic methods. We showcase the functionality of the framework with a comparative study against a deterministic algorithm on a PET dataset, with the use of the open-source Synergistic Image Reconstruction Framework (SIRF) [**?**].

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Learnable Priors Support Reconstruction in Diffuse Optical Tomography

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Keywords: Diffuse Optical Tomography; Generative model; Prior; Inverse problem; Neural network

Diffuse Optical Tomography (DOT) is a non-invasive medical imaging technique that makes use of Near-Infrared (NIR) light to recover the spatial distribution of optical coefficients in biological tissues for diagnostic purposes. Due to the intense scattering of light within tissues, the reconstruction process inherent to DOT is severely ill-posed [**?**]. In this work, we propose to tackle the ill-conditioning by learning a prior over the solution space using an autoencoder-type neural network. Specifically, the trained decoder part of the autoencoder is used as a generative model, that maps a latent code to estimated physical parameters given in input to the forward model [**?**]. The latent code is itself the result of an optimization loop which minimizes the discrepancy of the solution computed by the forward model with available observations. The structure and interpretability of the latent code is further enhanced by minimizing the rank of its covariance matrix, thereby promoting more effective utilization of its information-carrying capacity [**?**]. The deep learning-based prior significantly enhances reconstruction capabilities in this challenging domain, demonstrating the potential of integrating advanced neural network techniques into DOT.

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Applications of Geometric Function Theory to Inverse Problems

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Keywords: Geometric function theory; Layer potential technique; Analytic inversion

For a simply connected planar domain, the Riemann mapping theorem guarantees a unique conformal mapping from the exterior of the unit disk onto that of the domain. Geometric function theory connects the analytic properties of such conformal mappings to the geometric characteristics of the domain. Meanwhile, layer potential techniques have been extensively used to represent solutions of transmission problems in partial differential equations, yielding significant applications in inverse problems. This talk presents recent results that combine geometric function theory with layer potential techniques to address inverse problems, specifically the derivation of analytic inversion formulas for planar conductivity inclusions and the construction of neutral inclusions.

Some inverse problems in reconstructing cardiac valve geometry and heart fiber orientation

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Keywords: Inverse reconstruction problems; Carleman inequalities; cardiac valves; heart fiber orientation; liquid crystals

We present the mathematical analysis, modelling and simulation. of some inverse problems arising in medical imaging using partial differential equations. These problems involve the identification of cardiac valves and heart fibers. The first problem is related to the stability of non-invasively determining the geometry of cardiac valves from blood velocity measurements. This problem is reduced to the stable identification of zero-order potential coefficients in fluids. The second problem concerns the novel use of nonlinear models derived from liquid crystals to model and reconstruct cardiac fiber orientation.

The Journal of Inverse Problems and the Birth of Non-iterative Inversion

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Keywords: Inverse scattering; sampling methods; non-iterative inversion

The first paper on the linear sampling method in inverse scattering appeared in Inverse Problems in 1996 [**?**], marking the beginning of a new area in the field of inverse problems, known as non-iterative inversion approache. Over the past three decades, this area has experienced remarkable growth, giving rise to beautiful mathematics and paradigm-shifting strategies for solving nonlinear and ill-posed inverse problems. A vast body of literature has emerged, and the monographs [**?**, **?**, **?**] offer a glimpse into the diverse directions this research has taken. The journal Inverse Problems has played a central role in this development by disseminating groundbreaking research and fostering a vibrant international community.

In this talk, we begin with a quick overview of the field of non-iterative inversion methods, focusing primarily on linear sampling methods and interior eigenvalues in inverse scattering. Then we present some recent results on a spectral target signature for monitoring thin sheets of meta-material using electromagnetic waves [**?**]. In our analysis, the screen is represented by a central surface Γ, referred to as "screen", and its constitutive electromagnetic properties are modeled by a complex-valued matrix function defined on Γ, which is not necessarily sign-definite.

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Inverse Problems in the Age of AI

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Keywords: Inverse Problems; Artificial Intelligence; Deep Learning

The field of Inverse Problems has expanded from purely mathematical results, the development of algorithms and their computational implementation, the inclusion of statistical and Bayesian methods, to the ever rising trend to incorporate Artificial Intelligence (AI). This trend requires special attention beyond simply resorting to black box methods, because inverse problems usually involve complex physical models, very high dimensionality and scarcity of training data. In this talk I will review some of the main approaches for combining AI with Inverse Problems with an emphasis on explainabilty, scalability, and generalisability.

Sparse variational regularization with oversmoothing in the scale of sequence spaces

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Keywords: Linear ill-posed problem; sequence space; variational regularization; sparsity; oversmoothing

In this presentation, we consider a class of linear ill-posed problems, where the corresponding operator maps from the sequence space ℓ_τ ($\tau \geq 1$) into a Banach space and in addition satisfies a conditional stability estimate in the scale of sequence spaces ℓ_p, $p \geq 0$. For the regularization of this problem in the presence of deterministic noise, we consider Tikhonov regularization with a penalty functional either of the form $\mathcal{R} = \| \cdot \|_p^p$ for some $p > 0$ or $\mathcal{R} = \| \cdot \|_0$, this is, the counting measure. The latter case guarantees sparsity of the approximations. In this framework, we present convergence rates for a suitable a priori parameter choice. The results cover oversmoothing, i.e., the searched-for solution does not belong to the domain of definition of the considered penalty functional. The analysis of the latter case utilizes auxiliaries which are defined by means of hard thresholding.

Optimal rates of convergence for variational regularization of Poisson Inverse Problems

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Keywords: Inverse Problem; Poisson Data; Variational Regularization; Noise regularity; Poisson Process

In this talk we consider Inverse Problems

$$F(f) = g$$

with a possibly non-linear operator $F : D(F) \subset \mathcal{X} \to \mathbf{L}^1(M)$ for a bounded domain $M \subset \mathbb{R}^d$. We assume that the available data is a scaled Poisson process $G_t = \frac{1}{t}\sum_{i=1}^{N} \delta_{x_i}$ where the points x_i are drawn i.i.d. according to the density $tg^\dagger := tF(f^\dagger)$ with the unknown solution $f^\dagger \in \mathcal{X}$, $N \sim \mathrm{Poi}\left(t\int_M g^\dagger \,\mathrm{d}x\right)$ and $t > 0$ is an exposure time. Such problems occur e.g. in photonic imaging modalities such as super-resolution microscopy or coherent X-ray scattering.

To derive convergence rates of Tikhonov regularization

$$\hat{f}_\alpha \in \arg\min_{f \in D(F)} \left[\int_M F(f)\,\mathrm{d}x - \int_M \ln\left(F(f) + \sigma\right)\left(\,\mathrm{d}G_t + \sigma\,\mathrm{d}x\right) + \alpha\mathcal{R}(f)\right]$$

where $\sigma > 0$ is a user-specified offset, $\mathcal{R} : \mathcal{X} \to (-\infty, \infty]$ is a penalty term, and $\alpha > 0$ a regularization parameter, we apply a general approach from [1]. Therefore, we show that $G_t \in \mathrm{B}^{-d/2}_{2,\infty}(M)$ a.s., i.e. that the noise process G_t has the same regularity as a Gaussian white noise. This can be achieved via chaining. The other required assumptions such as a variational source condition and interpolation conditions are verified in examples.

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Convergence of Learnt, Unpaired Mappings for Inverse Problems

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Keywords: Regularization; Machine Learning, Generative Adversarial Networks, Unpaired Data, Optimal Transport

We present a framework for solving inverse problems via learnt, unpaired mappingsmodels trained to translate between measurement and solution spaces without requiring direct inputoutput correspondences. This regime imposes less restrictive data requirements than paired, supervised learning, yet sacrifices the robustness and convergence guarantees of classical inversion schemes. To bridge this gap, we introduce a CycleGAN-inspired training paradigm that enforces bidirectional consistency between forward and inverse operators. Under mild assumptions, we then prove the existence, stability, and convergence of the resulting mappings. Finally, we showcase numerical examples that demonstrate both the effectiveness and the robustness of our approach.

Geometry of Lebesgue-Bochner spaces with an application in dynamic inverse problems

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Keywords: Lebesgue-Bochner spaces; geometry of Banach spaces; Xu-Roach inequalities; Besov spaces; dynamic inverse problems

We consider time-dependent inverse problems in a mathematical setting using Lebesgue-Bochner spaces. Such problems arise when one aims to recover a function from given observations where the function or the data depend on time. Lebesgue-Bochner spaces allow to easily incorporate the different nature of time and space.

In this talk, we investigate geometrical properties of Lebesgue-Bochner spaces. In particular we compute the duality mapping and show that these spaces are smooth of power type in the same way as Besov spaces. To show this, we deliver the proof of a corollary of the Xu-Roach inequalities [1] for smoothness of power type.

In the end, we use these theoretical findings to implement Tikhononv regularization in Lebesgue-Bochner spaces using different regularities for time and space using the example of dynamic computerized tomography.

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The extended adjoint state and nonlinearity in correlation-based passive imaging

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Keywords: passive imaging, PDEs, correlation, covariance operator, tangential cone condition

This articles investigates physics-based passive imaging problem, wherein one infers an unknown medium using ambient noise and correlation of the noise signal. We develop a back-propagation framework via the so-called extended adjoint state, suitable for any linear PDE; crucially, this approach reduces by half the number of required PDE solves. Applications to several different PDE models demonstrate the universality of our method. In addition, we analyze the nonlinearity of the correlated model, revealing a surprising tangential cone condition-like structure, thereby advancing the state of the art towards a convergence guarantee for regularized reconstruction in passive imaging.

On Bayesian inference and OED in photoacoustic tomography with fractional attenuation

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Keywords: coefficient identification; uniqueness; Newton's method; acoustic nonlinearity parameter tomography; Jordan-Moore-Gibson-Thompson equation

In this talk we study the identification of the absorption density in photoacoustic tomography, formulated as the inverse problem of reconstructing either an initial condition or a space dependent source term in the wave equation. As relevant in ultrasound propagation, we incorporate power law frequency dependent attenuation by means of a time fractional damping term. We solve the inverse problem in a Bayesian framework using a Maximum A Posteriori (MAP) estimate, using an adjoint approach for gradient computation. On top of this, we consider optimization of the choice of the laser excitation function, which is the time-dependent part of the source in this model, to enhance the reconstruction result.

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Determination of a small elliptical conductivity anomaly from minimal and optimal boundary measurements

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Keywords: EIT; Optimal Experimental Design; Tikhonov Regularization

Our study considers the recovery of an elliptical conductivity anomaly within a homogeneous two-dimensional disk with minimal measurements. We consider two measurement strategies: (i) dipole-to-dipole measurements [1], and (ii) electrode-to-electrode measurements [2]. In (i) a dipole source is placed on the boundary and voltage gradient at the same location is measured. In (ii) a pair of electrodes are placed on the boundary and voltage drop between the electrodes are measured while current is driven from one electrode to the other. We solve the inverse problem of determining the parameters of the ellipse. For measurement (i), we place the dipole in five distinct locations (three for locating and sizing the ellipse). For (ii), we place four electrodes in distinct locations.

We investigate the issue of ill-posedness and optimal experiment design (OED). For the dipole measurement case, we explore both Bayesian and deterministic OED methods. In the case of electrode measurements, we stabilize the problem with partial Tikhonov regularization and improve the stability by OED. The main ideas are demonstrated in numerical experiments.

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Bayesian experimental design for head imaging by electrical impedance tomography

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Keywords: electrical impedance tomography, head imaging, Bayesian experimental design, A-optimality

Electrical impedance tomography is an imaging modality for deducing information about the conductivity inside a physical body from boundary measurements of current and voltage by a finite number of contact electrodes. This work considers the optimization of electrode positions in head imaging by electrical impedance tomography. The study is motivated by maximizing the sensitivity of electrode measurements to conductivity changes when monitoring the condition of a stroke patient, which justifies adopting a linearized version of the complete electrode model as the forward model. The algorithm is based on finding a (locally) A-optimal measurement configuration via gradient descent with respect to the electrode positions. The efficient computation of the needed derivatives of the complete electrode model is one of the focal points.

Enhancing MRI Head and Neck Imaging with GAN-Based Noise and Artifact Reduction

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Keywords: Generative Adversarial Networks; MRI images, Imege Denoising

This talk introduces an artificial intelligence approach based on a Generative Adversarial Network (GAN), designed to enhance the visual quality of magnetic resonance imaging (MRI) scans of the head and neck region. The proposed methodology demonstrates superior performance compared to existing state-of-the-art techniques. Experimental evaluations reveal that, once trained and validated, the model consistently improves image quality, particularly when applied to previously unseen, noise-corrupted data.

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From PET images to brain networks: a single-subject connectivity model

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Keywords: Brain Connectivity; Molecular PET imaging; Graph Theory; Divergence Metrics

Brain connectivity is a field of neuroscience focused on understanding how different brain regions are connected and interact with each other. Two main types of connections are typically distinguished: structural connectivity, which refers to the presence of anatomical links, and functional connectivity, which reflects temporal correlations in neural activity. These forms of connectivity are commonly studied through imaging modalities such as diffusion MRI, functional MRI, electroencephalography and magnetoencephalography. In both cases, the relationships between different brain regions are commonly represented as connectivity networks, where nodes correspond to brain regions and edges represent the presence or strength of a connection. From a mathematical point of view, these networks are modeled as graphs, providing a formal framework for analyzing the brain as a complex system [1, 2].

In this research, we propose a novel definition of brain connectivity derived from PET imaging data [4]. Starting from static PET images, we segment the brain into anatomical regions and, for each region, estimate a probability distribution modelling tracer uptake. We then compute distances between these distributions using several metrics - such as Kullback Leibler, Jensen Shannon and Hellinger - and evaluate their performance in capturing relevant relationships between different brain regions. Pairs of regions with more similar tracer distributions are assumed to be more strongly connected, allowing us to define edge weights as the reciprocal of these distances. This results in a weighted undirected graph representing a single-subject brain network, where each link reflects the degree of similarity in PET tracer uptake.

We explore the reliability of different metrics and the behaviour of the resulting graphs in relation to known patterns of PET metabolic activity is Alzheimer's disease - our field of application. Indeed, recent findings suggest that the architecture of brain networks plays a key role in shaping disease progression and clinical manifestation in Alzheimers disease [3].

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Mild data-driven regularization for brain stroke monitoring via microwave inverse scattering

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Keywords: Microwave inverse scattering, Data driven regularization, Banach spaces

Microwave inverse scattering is a non-ionizing imaging technique very useful in medical applications. In this regard, brain stroke microwave imaging (BSMI) is an emerging research area that has gained a lot of attention in the last few years. Indeed, BSMI potentially allows developing safe, relatively cheap, and portable systems, which can be used as complementary tools to perform an early diagnosis/differentiation (e.g., during patient transportation) or post-stroke continuous monitoring.

In the BSMI process, the quantitative reconstruction of the dielectric properties of the head involves the solution of an ill-posed, implicit, and non-linear integral equation. The specific imaging problem is however related to the identification of defects and inclusions inside a (partially) known structure. In such conditions, it is well known that some a-priori information about the structure and possible defects may allow a significant enhancement in the retrieved solution.

In this framework, a mild data-driven inversion technique is studied. Specifically, the approach relies upon the use of an inversion technique in variable-exponent Lebesgue spaces, in which an additional data-driven regularization term is added with the aim of including some information about the structure of the head and of the properties of the strokes into the inversion procedure. Such a regularization term, proposed in [1], exploits a simple surrogate model that is built from a set of training samples related to the host target only, and thus enforces a-priori information about it in a mild form. The effectiveness of the developed technique is assessed by numerical simulations involving a realistic head phantom derived from magnetic resonance images. As part of the DHEAL-COM project, this approach is useful to implement robust computational techniques in the case of proximity medicine applications.

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Calibrating parameters in mass-action chemical reaction networks through inverse problems

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Keywords: Chemical reaction networks; Parameters calibration; Inverse problems; Sensitivity analysis; Mass-action kinetics; Michaelis-Menten kinetics

Cell signaling processes are governed by complex networks of biochemical reactions involving numerous proteins. Mutations can disrupt these networks by altering the activity of specific chemical species, often leading to pathological conditions such as cancer. Chemical Reaction Networks (CRNs) based on mass-action kinetics offer a mathematical framework for modeling these systems through ordinary differential equations (ODEs), which describe the dynamics of protein concentrations within a single cell [1, 2]. Understanding CRN behavior in both physiological and mutated conditions is essential for identifying potential therapeutic strategies [3]. However, the building of accurate, cell dependent CRNs requires precise parameter values, which are often unavailable or difficult to measure directly.

The goal of this work is to calibrate the most sensitive parameters of a CRN by solving an inverse problem based on the system's dynamics. To this end, we propose a mathematical framework that combines different techniques, local sensitivity analysis [4] and model reduction among others, to estimate the parameters from equilibrium data [5]. This approach enables efficient parameter tuning, despite the challenges posed by the relatively small number of measurable species and the large quantity of unknowns.

As a first step of this calibration strategy, we solve a simple inverse problem, consisting in the reconstruction of microscopic rate constants for enzyme-substrate reactions from the corresponding Michaelis-Menten parameters. While these constants are more commonly available in the literature, mass-action kinetics instead requires knowledge of the underlying microscopic rate constants.

This modular approach contributes to the construction of a coherent and biologically meaningful CRN, laying the foundation for subsequent analyses. The work is carried out within the framework of the DHEAL-COM project, which aims to develop computational models to support decision-making in personalized cancer therapy.

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Learning the optimal regularizer for inverse problems

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Keywords: machine learning; regularization; linear inverse problems; sparsity

Variational regularization is a well-established technique to tackle instability of inverse problems, and it requires solving a minimization problem in which a mismatch functional is endowed with a suitable penalty term. The choice of such a functional is a crucial task, and it usually relies on theoretical suggestions as well as a priori information on the desired solution. A promising approach to this task is provided by data-driven strategies, based on statistical learning theory. In this talk, I will consider linear inverse problems (associated with relevant applications, e.g., in signal processing and in medical imaging), and aim at learning the optimal regularization operator, in a suitable sense.

I will first focus on the family of generalized Tikhonov regularizers, for which it is possible to prove theoretical properties of the optimal operator and error bounds for its approximation as the size of the sample grows, both with a supervised-learning strategy and with an unsupervised-learning one. Finally, I will discuss the extension to different families of regularization functionals, with a particular interest in sparsity-promotioning ones. This is based on joint work with G. S. Alberti, E. De Vito, S. Sciutto (University of Genoa), T. Helin (LUT University), M. Lassas (University of Helsinki) and L. Ratti (University of Bologna).

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Regularization via Latent Diffusion Models for Image Restoration

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Keywords: Diffusion Models; Latent Space; Inverse Problems; Image Restoration.

Motivated by recent advances in generative modeling, particularly the rise of Diffusion Models as the new state-of-the-art over Generative Adversarial Networks, we introduce Regularization by Latent Diffusion (RELD) [1]a novel algorithm for solving imaging inverse problems. RELD leverages the regularization properties of denoising processes, inspired by the Plug-and-Play [2] and RED [3] frameworks, by integrating a Latent Diffusion Model (LDM) trained for image denoising into a variational framework via Half-Quadratic Splitting (HQS). During training, the reverse diffusion process is conditioned on a universal encoder applied to noisy images; once trained, the LDM acts as a powerful generative prior within the variational model, where both the latent variables and conditioning inputs are optimized. This leads to an efficient two-step HQS-based optimization: one step with a closed-form solution and another solvable with a single gradient stepsignificantly reducing computational cost. RELD is validated on tasks such as denoising, deblurring, and super-resolution, and achieves competitive or superior performance compared to other state-of-the-art methods.

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Nonmonotone stochastic line searches without overhead for training neural networks

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Keywords: Large scale optimization; stochastic gradient descent; line search methods; neural networks

Gradient Descent (GD) and its stochastic variant (SGD) serve as the fundamental methods for training deep neural networks and solving various other inverse problems. At the same time, the speed of convergence and the robustness of these methods strongly rely on the selection of the step size. Despite the optimization doctrine would suggest using line searches to solve this problem, these methods have been neglected by the deep learning community because of the extra internal steps needed to find a suitable step size. In this talk, we will show that (nonmonotone) line search methods can often be applied without any internal overhead. More precisely, by adding some appropriate tests on the speed of convergence of the functions/gradients, the initial step size (e.g., Polyak) can be directly accepted without the extra costs of a backtracking technique. The theoretical analysis presented here shows that the method converges in the deterministic setting, while we propose some preliminary reasoning for the stochastic case. The numerical results illustrate that the new method maintains the same outer speed of convergence of the original one, while drastically reducing the number of backtracks.

Graph-Based Image Segmentation through Regularized Min-Cut Formulations

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Keywords: First keyword; Second keyword

This contribution addresses image segmentation as a typical ill-posed problem [1], approached through graph-based methods. We investigate regularized min-cut formulations by modeling the image as a weighted graph [2], where nodes represent pixels or superpixels and edges encode affinities. We also investigate regularized min-cut formulations on this structure. Indeed, standard graph-cut approaches, though efficient, often produce overly fragmented or biased segmentations due to the lack of global constraints. We introduce regularization terms that penalize small or unbalanced partitions to overcome these limitations and incorporate prior assumptions on region smoothness and connectivity. These regularizers are functionally analogous to Total Variation (TV) energies [3] in continuous settings and promote spatial coherence in the discrete domain. Additionally, we explore adaptive weighting schemes to modulate the cut cost according to local image contrast or texture.

The proposed approach is applied to the segmentation of prehistoric petroglyphsrock engravings characterized by faint edges, erosion, and high visual ambiguity. The regularized formulation proves crucial in guiding the segmentation process toward meaningful solutions, especially when low-level cues are weak or conflicting. Experimental results demonstrate that integrating variational regularization significantly improves segmentation robustness in such challenging archaeological imagery.

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Learning Optimal Filters Using Variational Inference

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Keywords: Filtering; Variational Inference; Data Assimilation; Probabilistic Estimation; Analysis Map

Filtering—the task of estimating the conditional distribution for states of a dynamical system given partial and noisy observations—is important in many areas of science and engineering, including weather and climate prediction. However, the filtering distribution is generally intractable to obtain for high-dimensional, nonlinear systems. Filters used in practice, such as the ensemble Kalman filter (EnKF), provide biased probabilistic estimates for nonlinear systems and have numerous tuning parameters. Here, we present a framework for learning a parameterized analysis map—the transformation that takes samples from a forecast distribution, and combines with an observation, to update the approximate filtering distribution—using variational inference. In principle this can lead to a better approximation of the filtering distribution, and hence smaller bias. We show that this methodology can be used to learn the gain matrix, in an affine analysis map, for filtering linear and nonlinear dynamical systems; we also study the learning of inflation and localization parameters for an EnKF. The framework developed here can also be used to learn new filtering algorithms with more general forms for the analysis map.

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Tuning randomized learning algorithms: A case study for random feature regression

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Keywords: Random features; Gaussian process regression; hyperparameter learning; ensemble Kalman inversion, Bayesian inverse problems

Randomized algorithms exploit stochasticity to reduce computational complexity. One important example is random feature regression (RFR) that accelerates Gaussian process regression (GPR). RFR approximates an unknown function with a random neural network whose hidden weights and biases are sampled from a probability distribution. Only the final output layer is fit to data. The hyperparameters that characterize the sampling distribution greatly impact performance, yet are not directly accessible from samples. This makes optimization of hyperparameters via standard (gradient-based) optimization tools inapplicable. Inspired by Bayesian ideas from GPR, we introduce a random objective function that is tailored for hyperparameter tuning of random features. The objective is minimized with ensemble Kalman inversion (EKI), a gradient-free particle-based optimizer that is scalable to high-dimensions and robust to randomness in objective functions. We illustrate the performance of these tools on examples whose success require good off-training-data performance. The success of the proposed EKI-based algorithm for RFR suggests its potential for automated optimization of hyperparameters arising in other randomized algorithms, while the black-box nature of the optimization allows it to easily adopt new loss functions arising from the latest kernel literature.

The main body of work is published [1]; and open-source, registered, and maintained Julia-language software is available [2, 3, 4]

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Stochastic interpolants: from generative modeling to generative science and engineering

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Schrödinger Bridge Flow for Unpaired Data Translation

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Inexact Newton regularizations with uniformly convex stability terms: A unified convergence analysis

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Keywords: Inexact Newton regularization; Convex penalization; Regularization techniques.

We present a unified convergence analysis of inexact Newton regularizations with general uniformly convex penalty terms for nonlinear ill-posed problems in Banach spaces. These schemes consist of an outer (Newton) iteration and an inner iteration, which provides the update of the current outer iterate. To this end, the nonlinear problem is linearized about the current iterate and the resulting linear system is inexactly solved by an inner regularization method. In our analysis, we only rely on generic assumptions of the inner methods, and we show that a variety of regularization techniques satisfies these assumptions. For instance, gradient-type and iterated-Tikhonov methods are covered. Not only the technique of proof is novel, but also some results, because for the first time uniformly convex penalty terms stabilize the inner scheme in full generality. Numerical experiments based on the inverse problem of electrical impedance tomography illustrate the impact of different uniformly convex penalty terms.

Regularization techniques for tomographic data preprocessing

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Keywords: First keyword; Second keyword

In this work, we present a computational imaging discussion on the classical Paganin approach for recovering the phase of X-ray tomographic measurements using the Transport-of-Intensity Equation (TIE) with a single-material parameter. In practical experiments, this parameter often carries significant uncertainty, leading tomographic users to iteratively test various values until a reconstruction provides useful information. From a computational perspective, this trial-and-error process affects user experience, as it is both time-consuming and resource-intensive. We compare the classical formulation of the problem in the projection domain (also referred to as the frame domain) with an alternative filtering strategy in the sinogram domain (also referred to as the slice domain). Additionally, we propose bounds to quantify the differences between these approaches. Processing directly in the slice domain is significantly faster, as the phase retrieval is integrated with the reconstruction process, effectively combining two steps into one. This integration enhances the user experience in tomographic beamline settings.

Tomographic experiments at beamlines frequently use the Paganin regularization [1] to recover phase via the Transport-of-Intensity Equation (TIE) [2]. This regularization depends on a parameter λ, typically chosen based on material properties, requiring users to test multiple values. In its original formulation [1], the filter is applied to projections (radiographs/frames). However, working with frames involves accessing data along the *slow* direction, which is time-consuming in large-scale tomographic experiments. In [3], an alternative approach applied regularization along the *fast* direction, processing the volume slice by slice. This slice-wise filtering aligns with tomographic reconstruction filters, significantly accelerating parameter testing. Consequently, the Paganin process can be applied either frame-wise or slice-wise. This manuscript demonstrates that the optimal λ in slice-wise filtering is approximately a scaled version of the frame-wise value, with the scaling factor being sample-dependent.

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On a regularization strategy for constrained inverse problems

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Keywords: Regularization; Constrained, Inverse Problems, Applications

In this presentation, our focus is on finding a regularized solution for a constrained problem that is ill-posed and can be expressed as

$$F(P(\phi)) = y^\delta\,, \tag{1}$$

$$K(\phi) = 0\,, \tag{2}$$

where F, P, and K represent potential nonlinear operators acting within suitable Banach spaces, with measured data y^δ satisfying

$$\|y - y^\delta\| \le \delta\,. \tag{3}$$

We demonstrate that, given certain assumptions regarding the operators F, P, and K, the augmented Lagrangian

$$L_\alpha(\phi; (\lambda, \mu); y^\delta) := \|F(P(\phi)) - y^\delta\| + R_\alpha(\phi, \phi_0) + \langle \lambda, K(\phi)\rangle + \mu\|K(\phi)\|, \tag{4}$$

where the pair (λ, μ) are generalized Lagrangian multipliers, R_α is a regularization term and ϕ_0 is the a-priory guess, has no duality gaps. Furthermore, its primal solution

$$\phi_\alpha^\delta := \phi_\alpha^\delta(\lambda_\alpha^\delta, \mu_\alpha^\delta) = argmin_\phi L_\alpha(\phi; (\lambda, \mu); y^\delta) \tag{5}$$

is a regularized solution [1] for the problem (1).

The regularization method is utilized for the simultaneous identification of absorption and scattering coefficients in a two-dimensional optical tomography problem [2].

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On an inertial Levenberg-Marquardt method

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Keywords: Ill-posed problems; Inertial methods; Levenberg Marquardt method.

For standard inverse problem [1], modeled by an ill-posed equation

$$F(x) \ = \ y^\delta, \tag{1}$$

where $F : X \to Y$ is a nonlinear, Fréchet differentiable, ill-posed operator. We recall a family of implicit iterative type methods for obtaining stable approximate solutions to nonlinear ill-posed type operator equations as in (1). The Levenberg-Marquardt (LM) type methods are defined by

$$x_{k+1}^\delta := \operatorname{argmin}_x \left\{ \|F(x_k^\delta) + F'(x_k^\delta)(x - x_k^\delta) - y^\delta\|^2 + \lambda_k \|x - x_k^\delta\|^2 \right\}, \quad k = 0, 1, \ldots \tag{2a}$$

$$\left(A_k^* A_k + \lambda_k I\right)(x - x_k^\delta) = A_k^* \left(y^\delta - F(x_k^\delta)\right), \quad k = 0, 1, \ldots \tag{2b}$$

where $A_k := F'(x_k^\delta) : X \to Y$ is the Fréchet derivative of F evaluated at x_k^δ and $A_k^* : Y \to X$ is the adjoint operator to A_k. Additionally, (λ_k) is a positive sequence of Lagrange multipliers. The iteration starts at a given initial guess $x_0 \in X$.

We analyze an inertial method that consists in choosing appropriate non-negative sequences (α_k), (λ_k) and defining the extrapolation $w_k^\delta := x_k^\delta + \alpha_k(x_k^\delta - x_{k-1}^\delta)$, if where $x_{-1} = x_0 \in X$ are given, the next iterate x_{k+1} is then defined by

$$x_{k+1}^\delta := \operatorname{argmin}_x \left\{ \|F(w_k^\delta) + F'(w_k^\delta)(x - w_k^\delta) - y^\delta\|^2 + \lambda_k \|x - w_k^\delta\|^2 \right\}, \quad k = 0, 1, \ldots \tag{3}$$

Our approach for inverse problems is connected to the inertial method put forth in [2]. We also provide applications for identification problems in elliptic PDEs and in machine learning for the method in (3). In the case of linear ill-posed operator equations, an approach analogous to the one addressed in [3] is followed.

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Recovery of Time-Dependent Coefficients in Hyperbolic Equations on Manifolds

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Keywords: time-dependent coefficients; wave equation; Carleman estimate; partial data; Riemannian manifolds

In this talk we discuss inverse problems of determining time-dependent lower order coefficients appearing in the wave equation of a compact Riemannian manifold of dimension three or higher. More specifically, we are concerned with the case of conformally transversally anisotropic manifolds, or in other words, compact Riemannian manifolds with boundary which are conformally embedded in a product of the Euclidean line and a transversal manifold.

With an additional assumption of the attenuated geodesic ray transform being injective on the transversal manifold, we prove that the knowledge of a certain partial Cauchy data set determines some time-dependent lower order coefficients of the hyperbolic equation uniquely. We shall discuss two problems: (1) Recovery of a time dependent potential appearing in the wave equation, when Dirichlet and Neumann values are measured on opposite parts of the lateral boundary of the space-time cylinder. (2) Recovery of both time-dependent damping coefficient and potential, when the Dirichlet values are measured on the whole lateral boundary and the Neumann data is collected on roughly half of the boundary.

An inverse problem for a nonlinear biharmonic operator

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Keywords: Nonlinear biharmonic operator, Runge approximation

An inverse problem for a nonlinear biharmonic operator is under consideration in the spirit of [1, 2]. We prove that a general nonlinear term of the form $Q = Q(x, u, \nabla u, \Delta u)$ associated to a nonlinear biharmonic operator can be recovered from the local Cauchy data set. The proof uses first order linearization method, Runge approximation, and uniqueness results for the linearized inverse problem. This is a joint work with Janne Nurminen from the University of Jyväskylä.

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Fast coefficient identification in periodic reaction-diffusion-advection models

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Keywords: Inverse problem; Asymptotic expansion method, Singularly perturbed partial differential equations

This work addresses both direct simulation and inverse parameter recovery for reaction-diffusion-advection equations with time-periodic forcing, in which a small diffusion parameter produces a sharp moving front. We first derive a uniformly valid periodic solution by matched asymptotic expansions, capturing the inner transition layer and proving rigorous error bounds for the solution profile and front location. Building on these expansions, we introduce two simple recovery formulasone for spatially varying coefficients and one for time-varying coefficientsthat bypass full PDE-constrained optimization. Each formula reduces the inverse problem to a regularized least-squares fit of easily measured quantities, and we prove stability and root-noise convergence under realistic measurement errors. Numerical experiments confirm that our method reconstructs unknown coefficients accurately and efficiently, even in the presence of moderate noise, offering a practical alternative to traditional inverse-problem techniques in environmental, biological, and engineering applications.

Source function recovery with asymptotic method for three-dimensional Reaction-Diffusion-Advection equations

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Keywords: Asymptotic expansion method; inverse problem; singular perturbed partial differential equations

In this study, our primary goal is to recover the source function of the dynamics of the moving front in the case of three-dimensional differential problems. Typically, such a process could be observed as a result of in situ combustion during oil production, or various autowave models and the propagation of acoustic waves.

Our analysis is focused on singularly perturbed reaction-diffusion-advection type initial-boundary-value problem of a general form. We employ methods from asymptotic theory to develop an approximate smooth solution with an internal layer. The core idea of this analysis is the usage of the matching asymptotic expansion method. Using local coordinates, we focus on the transition layer, where the solution undergoes rapid changes. Once the location of the transition layer is established, we can approximate the solution across the entire domain of the problem depending on the accuracy we desire. Next, we incorporate this approach into the numerical algorithm for recovering the source function. Numerical examples with error estimations are provided in order to demonstrate the high accuracy of the asymptotic method in predicting the behaviors of moving fronts.

Diffuse optical tomography with an imperfectly known boundary

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Keywords: Diffuse optical tomography; inaccurately known boundary shape; conformal deformation

Diffuse optical tomography (DOT) is a non-invasive medical imaging modality in which images of the optical properties of a biological tissue are derived based on measurements of near-infrared light on the surface of the body. In many practical applications one often lacks exact knowledge of the domain boundary. This is problematic as even small errors in the boundary shape of the computation domain may lead to large artifacts in the reconstructed images. A method is proposed that simultaneously reconstructs the optical properties (diffusion and absorption coefficients) and the boundary shape in DOT. The approach consists of three steps: first, an anisotropic diffusion and an absorption coefficient reproducing the measured DOT data in a model domain are computed. Second, using isothermal coordinates, it is possible to construct a deformation that makes the diffusion isotropic and both this coefficient and the absorption coefficient turn out to be a conformally deformed image of the true coefficients in the exact domain. The final step involves minimizing the error between certain known geometric properties (such as the perimeter) corresponding to the exact domain and the reconstructed domain, again through conformal deformations. The method is implemented and applied to simulated noisy data, demonstrating that simultaneous recovery of the diffusion coefficient, the absorption coefficient and the boundary shape is feasible in DOT.

Lebesgue-Bochner spaces in dynamic inverse problems

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Keywords: dynamic inverse problems; Lebesgue-Bochner spaces; geometry of Banach spaces; regularization; computerized tomography

We consider time-dependent inverse problems in a mathematical setting using Lebesgue-Bochner spaces. Such problems arise when one aims to recover a function from given observations where the function or the data depend on time. Lebesgue-Bochner spaces allow to easily incorporate the different nature of time and space.

In this talk, we present two different regularization methods in Lebesgue Bochner spaces:

1. classical Tikhonov regularization in Banach spaces
2. variational regularization by penalizing the time-derivative

In the first case, we investigate geometrical properties of Lebesgue Bochner spaces. In particular we compute the duality mapping and show that these spaces are smooth of power type. With this we can implement Tikhononv regularization in Lebesgue-Bochner spaces using different regularities for time and space.

In the second case, we develop an algorithm to solve the minimization problem

$$\min_{\vartheta\in\mathbb{X}} E(\vartheta) = \min_{\vartheta\in\mathbb{X}} \left\{ \frac{1}{2}\|A\vartheta-\psi\|^2_{L^2(0,T,L^2(\Omega))} + \frac{\alpha}{2}\|\vartheta\|^2_{L^2(0,T,L^2(\Omega))} + \frac{\beta}{2}\|\partial_t\vartheta\|^2_{L^2(0,T,H^{-1}(\Omega))} \right\},$$

where

$$\mathbb{X} = L^2(0,T,L^2(\Omega)) \cap H^1(0,T,H^{-1}(\Omega)),$$

T is the end time, Ω the spatial domain, A the dynamic forward operator, ϑ the data and α, β are regularization parameters.

We test both methods using the example of dynamic computerized tomography.

Vectorial Total Variation for Image Reconstruction in Spectral Photon-Counting Detector CBCT

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Keywords: Vectorial total variation; Tomography; Cone beam computed tomography; Photon-counting detectors

Photon-counting detector cone beam computed tomography (PCD-CBCT) is a new paradigm of tomographic imaging that utilizes energy-sensitive X-ray detectors. Compared to conventional cone beam CT, PCD-CBCT offers many benefits, including single-acquisition spectral imaging, and improved soft-tissue contrast and Hounsfield unit (HU) accuracy. PCD-CBCT prototype imaging solutions have already been developed, and the technology will most likely be adapted into clinical imaging in the near future.

In spectral PCD imaging, the detectors divide the detected X-ray photons into multiple separate energy bins. The limited number of photons collected in each bin results in poor signal-to-noise ratio (SNR). Consequently, traditional analytical reconstruction methods such as FBP and FDK may result in suboptimal reconstruction quality, and more advanced reconstruction techniques are needed.

In this work, we developed reconstruction algorithms for PCD-CBCT based on vectorial total variation (VTV) regularization, an extension of the widely used total variation regularization scheme into multichannel data. Three different approaches were used: channel-by-channel total variation (CBC-TV), total Frobenius variation (TFV), and total nuclear variation (TNV). The algorithms utilize an efficient primal-dual fixed point (PDFP) optimization algorithm, and they were tested and benchmarked on real data collected using an in-house constructed PCD-CBCT imaging setup.

Efficient Dynamic Image Reconstruction with Motion Estimation

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Keywords: Dynamic inverse problems, generalized Krylov subspace, computerized tomography, motion estimation

Dynamic inverse problems are challenging to solve due to the need to identify and incorporate appropriate regularization in both space and time. Moreover, the very large scale nature of such problems in practice presents an enormous computational challenge. In this work, in addition to the use of edge-enhancing regularization of spatial features, we propose a new regularization method that incorporates a temporal model that estimates the motion of objects in time. In particular, we consider the optical flow model that simultaneously estimates the motion and provides an approximation for the desired image, and we incorporate this information into the cost functional as an additional form of temporal regularization. We discuss a computationally efficient algorithm to solve the jointly regularized problem that leverages a generalized Krylov subspace method. We illustrate the effectiveness of the prescribed approach on a wide range of numerical experiments, including limited angle and single-shot computerized tomography.

Modified error-in-constitutive-relation framework for the characterization of linear viscoelastic solids

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Keywords: Elastography; dissipative solids

We develop an error-in-constitutive-relation (ECR) approach [1] toward the full-field characterization of heterogeneous viscoelastic solids described as generalized standard materials [2]. To this end, we formulate the linear viscoelastic behavior in terms of the Helmholtz free energy potential and a dissipation potential. Assuming the availability of full-field interior kinematic data, the constitutive mismatch between the kinematic quantities (strains and internal thermodynamic variables) and their flux counterparts (Cauchy stress tensor and that of thermodynamic tensions), commonly referred to as the ECR functional, is established with the aid of Legendre-Fenchel gap functionals linking the thermodynamic potentials to their energetic conjugates. We then proceed by introducing the modified ECR (MECR) functional as a linear combination between the (ECR) constitutive mismatch E and the kinematic data misfit M, weighted by a regularization coefficient κ. On demonstrating that the $M(\kappa)$ vs. $E(\kappa)$ plots a convex arc, we identify κ via the L-curve approach [3]. The affiliated stationarity conditions then yield two coupled evolution problems, namely (i) the forward evolution problem for the (trial) displacement field driven by the constitutive mismatch, and (ii) the backward evolution problem for the adjoint field driven by the data mismatch. This allows us to establish compact expressions catering for minimization of the MECR functional. For generality, the formulation is established assuming both time-domain (i.e. transient) and frequency-domain data. We illustrate the developments in a two-dimensional (2D) setting by pursuing the multi-frequency MECR reconstruction of (i) piecewise-homogeneous standard linear solid, and (b) smoothly-varying Jeffreys viscoelastic material. A preliminary 1D application of the proposed methodology toward exposing the mechanical fingerprint of reaction-driven microcrackng and pore filling in rod-shaped rock specimens will also be discussed.

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Asymptotics Analysis Applied to Small Volume Inverse Shape Problems

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Keywords: Diffuse Optical Tomography ; Inverse Scattering ; MUSIC Algorithm ; Direct Sampling

In this talk, we consider two inverse shape problems coming from diffuse optical tomography and inverse scattering. In both problems, our goal is to reconstruct small-volume interior regions from measured data on the exterior surface of an object. In order to achieve this, we will derive an asymptotic expansion of the reciprocity gap functional associated with each problem. The reciprocity gap functional takes in the measured Cauchy data on the exterior surface of the object. In diffuse optical tomography, we prove that a MUSIC-type algorithm can be used to recover the unknown subregions. For the problem coming from inverse scattering, we recover the subregions of interest via a direct sampling method. We will present numerical examples for both cases in two dimensions, where the measurement surface is the unit circle.

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On the instabilities of naive FEM discretizations for PDEs with sign-changing coefficients

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Keywords: sign-changing coefficients; meta materials; finite element method; stability analysis

Artificially produced meta materials offer remarkable prospects to control waves and to construct cloaking devices. When simulating such materials the respective multi scale structure needs to be dealt with, either by an expensive mesh resolution or by problem adapted finite element methods. A more simpler and convenient approach is to consider the fully homogenized model, which avoids the multiscale structure, but leads to partial differential equations with sign-changing coefficients. The highly indefinite nature of such PDEs calls for the development of problem adapted FEMs to ensure reliable simulations, and several such approaches have been proposed [2, 3, 4]. However, in numerical experiments the suspected instabilities of standard FEMs often do not manifest themselves in a conclusive manner, raising the question on the necessity of specialized methods.

To shed light on the former phenomenon we construct an explicit example with a broad family of meshes for which we prove that the corresponding naive finite element discretizations are unstable. On the other hand, we also provide a broad family of (non-symmetric) meshes for which we prove that the discretizations are stable. Together, these two findings explain the results observed in numerical experiments.

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Nonlinear Rytov approximation with the inverse Rytov series

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Keywords: Inverse diffusion problems; Optical tomography; Inverse series

We consider the following diffusion equation.

$$\begin{cases} -D_0\Delta u + \mu_a u = f, & x \in \Omega \\ D_0\partial_\nu u + \frac{1}{\zeta}u = 0, & x \in \partial\Omega, \end{cases}$$

where D_0, ζ are positive constants and ∂_ν is the directional derivative with the outward unit vector ν normal to $\partial\Omega$. We assume that the absorption coefficient μ_a is given by

$$\mu_a(x) = (1 + \eta(x))\, g,$$

where $g > 0$ is a constant. By the inverse Rytov series we can obtain η as [1]

$$\eta = \mathcal{J}_1\psi + \mathcal{J}_2\psi \otimes \psi + \cdots,$$

where

$$\psi(x) = \ln \frac{u_0(x)}{u(x)}, \quad x \in \partial\Omega.$$

Here, u_0 is the solution to the above-mentioned diffusion equation with $\eta \equiv 0$.

Recently, the inverse Rytov series was extended to the time-dependent diffusion equation and the optical tomography of finding η with the inverse Rytov series was verified using a phantom experiment [2].

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Tensor Tomography on the Hyperbolic Disk

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Keywords: X-ray transform; hyperbolic space

The geodesic X-ray transform acting on functions or symmetric tensor fields is a central object in geometric inverse problems. In this talk, we will focus on the X-ray transform in the classical geometric setting of two-dimensional hyperbolic space. This setting is particularly interesting due to its lack of compactness and its role as a model for more general geometries, such as asymptotically hyperbolic spaces. We will report on recent progress in characterizing the range of the geodesic X-ray transform on functions and symmetric tensors in this setting, deriving singular value decompositions, establishing sharp mapping properties, and developing reconstruction procedures.

Inversion formula and UCP results for the mixed ray transform in 2-D Euclidean space

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Keywords: Inversion formula; Unique continuation results; Mixed ray transform

This talk revolves around various aspects of the mixed ray transform of $(k + \ell)$-ordered tensor fields that are symmetric in its first k and last ℓ indices. We start with an algorithm to recover the solenoidal part of the unknown tensor field using the normal operator of the mixed ray transform. Next, we establish a set of unique continuation results. This is based on a joint work with Rohit Kumar Mishra and Suman Kumar Sahoo.

2D V-line Tensor Tomography

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Keywords: V-line transform; Tensor tomography; Inversion algorithms; Integral moments

The reconstruction problem of tensor fields has been studied in many classical works of integral geometry. The underlying problem involves the inversion of integral transforms of unknown tensor fields, including the longitudinal (Doppler) ray transform, mixed ray transform, transverse ray transform, and momentum ray transform of different orders. We consider a generalization of these transforms, in which the linear integration path is replaced either by V-lines (broken rays) or stars (a finite union of rays emanating from a common vertex). We first focus on the case of vector fields and present several exact closed-form inversion formulas along with some numerical simulations. Then we move to the case of symmetric 2-tensor fields and discuss recent developments in the area.

Manifold learning and inverse problems

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Keywords: Manifold learning, inverse problems, wave equation, heat equation

We consider the manifold learning problems in Euclidean and invariant settings. In the Euclidean manifold learning, the problem is to construct a submanifold close to a smooth submanifold $S \subset \mathbb{R}^d$ when one is given the points $y_j = x_j + \epsilon_j$, $j = 1, 2, \ldots, N$, where x_j randomly sampled points of S and ϵ_j are noise vectors, for example, independent Gaussian noise. This problem in encountered in Cryo-electron microscopy that is used to reconstruct the structure of viruses and proteins. In the invariant manifold learning the task is to construct a Riemannian manifold that approximates a given discrete metric space. This problem is encountered in medical and seismic imaging where an unknown wave speed in a domain needs to be determined from indirect measurements. In geometric terms, this corresponds to the reconstruction of the Riemannian metric associated with the wave velocity from the wave kernel (or the heat kernel) measured in a subset of the domain.

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On the identification of early tumor states in some nonlinear reaction-diffusion models

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Accurately learning a tumour's initial spatial configuration from a single late-time image would unlock better calibration of patient-specific growth models and improve treatment planning. I will present two recent advances in model-constrained inverse problems that address this challenge with diffuse-interface phase-field equations. I will first review a study on prostate cancer, where a nutrient-driven phase-field model coupled to a PSA reaction–diffusion equation yields a highly ill-posed backward problem. We show well-posedness, prove continuous Fréchet differentiability of the solution map, and obtain only logarithmic stability in general, but optimal Lipschitz stability once the unknowns are confined to a finite-dimensional manifold, opening the door to locally convergent Landweber iterations.

I will then focus on a coupled Cahn–Hilliard / reaction-diffusion model for avascular growth and prove uniqueness of the backward reconstruction under minimal regularity, and set up an adjoint-based Tikhonov framework whose finite-element implementation accurately reconstructs 2-D/3-D initial states from noisy final data. The talk emphasises the mathematical relevance of diffuse interface models in tumor growth, the role of logarithmic convexity in backward uniqueness and stability, and the practical impact of interface-energy regularisation on numerical performance.

Regularization of Nonlinear Inverse Problems - From Functional Analysis to Data-Driven Approaches

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The journal Inverse Problems has been founded 40 years ago. When the journal turned 25 some publications were flagged as outstanding. One of them was the paper of Engl, Kunisch, Neubauer "Convergence rates for Tikhonov regularization of nonlinear ill–posed problems", Inverse Problems, 5.3 (1989), 523-540. The analysis of this paper is based on functional analysis. We use their fundamental techniques to analyze Tikhonov regularization which is based on data-driven approaches.

With the Journal Inverse Problems: a personal viewpoint

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Keywords: uniqueness, stability, time-fractional partial equation

In the Journal Inverse Problems, I have published more than 80 articles with my colleagues and my first article is Engl, Scherzer and Yamamoto [1] in 1994. Since then, my publication list can indicate a cross section of my theoretical research activities on inverse problems for partial differential equations. I will pick up some of such articles to give an overview:

- Stability and uniqueness for inverse coefficient problems for parabolic and hyperbolic equations by Carleman estimates: [2] - [3], [6]
- The uniqueness for the inverse obstacle problem: [4]
- Dirichlet-to-Neumann maps: [7]
- Invrse problem for time-fractional diffusion equation [5]

I am not only restricted to any retrospects, but also I would like to mention future hopeful topics mainly concerning inverse problems for non-local partial differential equation such as fractional partial differential equations.

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Convex regularization for static and dynamic inverse problems with randomly subsampled measurements

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Keywords: Convex regularization; Random measurements; Dynamic tomography; Besov spaces; Shearlets

In several challenging application scenarios, one has to reconstruct an unknown quantity from indirect, noisy measurements, which are moreover affected by a random subsampling process. A prominent example of such a task is represented by X-ray tomography with randomly sampled angular views.

In this talk, I will present a comprehensive approach to the regularization of these problems, described in [1]. Leveraging tools and techniques from both statistical learning and inverse problems, I will derive convergence estimates of the regularized solution to the ground truth, both as the noise on the data reduces and as the number of evaluation points increases.

I will focus on a class of convex, p-homogeneous regularization functionals (p ranging between 1 and 2), motivated by the growing interest towards sparsity-promoting techniques. Applying the general results in different functional spaces, such as Besov or smoothness spaces, our approach is able to encompass a large variety of sparsifying transforms. I will first showcase its application to wavelet-based regularization, extending it to the use of shearlet frames and finally to cylindrical shearlets, a recently introduced tool specifically suited for dynamic problems (see [2]).

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How to learn to regularize variationally

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Keywords: Variational regularization, Tikhonov regularization, supervised learning

We investigate the problems of learning variational regularization in a supervised context. Our aim is to understand the problem theoretically and hence, we focus on simple linear finite dimensional linear problems and basic regularization methods like Tikhonov, quadratic (not necessarily convex) and Lavrentiev regularization. Our aim is to understand when and how learning of operators and priors works. Especially we derive the respective optimal methods for given distributions for the solutions and the noise. We will identify conditions on these distributions under which the above approaches will lead to the same result and investigate how the results differ otherwise.

A unified variational analysis of non-standard noise models

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Keywords: variational regularization; variational source conditions; statistical inverse problems; large noise; operator approximations

In much of the literature on regularization theory, errors in the data are simplistically modeled by an (infinite-dimensional) vector in the image space of the forward operator or by (Gaussian) white noise. Papers on more realistic noise models often consider rather special settings concerning the forward operator and/or the smoothness of the exact solution. In this talk we present a unified framework for the analysis of general, more realistic noise models in variational regularization. The central assumption is called a variational noise bound, and it naturally complements the well-studied variational source conditions describing the smoothness of the exact solution. We show that our assumptions imply existence of minimizers, the proof of which can be a tedious task in case of large noise where the data term is not bounded from below. We show that our setting can be used to analyze discrete data, Gaussian white noise, Poisson noise, errors in the operator, and impulsive noise. In cases where lower bounds are known our error bounds are of optimal order. In several cases previous proofs are considerably simplified.

Stochastic Gradient Methods in Banach Spaces

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Keywords: Stochastic Gradient Methods; Inverse Problems; Banach Spaces; Regularisation

In this work, we develop a novel mathematical framework and provide a rigorous analysis of Stochastic Gradient Descent (SGD) in Banach spaces, targeting a class of linear and nonlinear inverse problems. The extension of SGD analysis to the Banach setting introduces unique theoretical challenges, necessitating new mathematical tools. To address these, we integrate techniques from Hilbert space theory with modern optimization strategies. Alongside the theoretical contributions, we present numerical results that demonstrate the practical performance and potential of our approach. Our findings pave the way for broad applications, and we outline key challenges and promising directions for future research.

On Qualitative Experimental Design

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Keywords: Experimental design; Sensitivity Analysis; Matrix Sketching; Bacterial Chemotaxis

Experimental design often has a strong effect on the well- or ill-posedness of an inverse problem, as it determines the quality of available data. Classical methods of optimal experimental design rephrase the task of finding an experimental design that yields informative data on the inversion quantity as an optimization problem that minimizes uncertainty of the reconstruction. In this talk, we want to shed light on another perspective to experimental design: qualitative experimental design relaxes the necessity to find an *optimal* design w.r.t. some criterion, and instead concentrates on finding a *suitable* design that yields data that is sensitive w.r.t. the parameter and thus facilitat inverse problem - at least locally around a presumed ground truth. This opens a new methodological toolbox, and we lay out two strategies for PDE parameter identification:

- the relaxation-of-theory approach handcrafts an experimental design based on a discretization of a suitable constructive theoretical existence and uniqueness proof. It is demonstrated on the inverse problem related to the biological phenomenon of bacterial chemotaxis.
- a generally applicable sampling approach based on matrix sketching of the sensitivity / Fisher information matrix of the linearized system. This method shall be demonstrated on an easy toy system, the potential reconstruction problem related to the stationary Schrödinger equation.

Optimal approximation error approach in Bayesian inverse problems

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Keywords: Bayesian inverse problems; surrogate modelling; experimental design

In optimal experimental design (OED), the goal is to design experiments that maximize the information gained about unknown parameters. In many problems, evaluating the forward operator is computationally expensive, making it common practice to replace it with a less costly approximate model. However, such approximations introduce errors that can distort the resulting posterior distribution. We focus on the *approximation error model* (AEM), which accounts for the modelling error by incorporating it into the observational noise. A central challenge in this approach is how to choose the modified noise covariance in a principled way. To address this, we propose a new framework which seeks to minimize the distance between the true and approximate posterior distributions. We relate our method to stability concepts in OED.

TAEN: A Model-Constrained Tikhonov Autoencoder Network for Forward and Inverse Problems

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Keywords: Inverse Problems; Randomized data

Efficient real-time solvers for forward and inverse problems are essential in engineering and science applications. Machine learning surrogate models have emerged as promising alternatives to traditional methods, offering substantially reduced computational time. Nevertheless, these models typically demand extensive training datasets to achieve robust generalization across diverse scenarios. While physics-based approaches can partially mitigate this data dependency and ensure physics-interpretable solutions, addressing **scarce data regimes** remains a challenge. Both purely data-driven and physics-based machine learning approaches demonstrate severe overfitting issues when trained with insufficient data. We propose a novel Tikhonov autoencoder model-constrained framework, called TAE, capable of learning both forward and inverse surrogate models using a single arbitrary observation sample. We develop comprehensive theoretical foundations including forward and inverse inference error bounds for the proposed approach for linear cases. For comparative analysis, we derive equivalent formulations for pure data-driven and model-constrained approach counterparts. At the heart of our approach is a data randomization strategy, which functions as a generative mechanism for exploring the training data space, enabling effective training of both forward and inverse surrogate models from a **single observation**, while regularizing the learning process. We validate our approach through extensive numerical experiments on two challenging inverse problems: 2D heat conductivity inversion and initial condition reconstruction for time-dependent 2D Navier-Stokes equations. Results demonstrate that TAE achieves accuracy comparable to traditional Tikhonov solvers and numerical forward solvers for both inverse and forward problems, respectively, while delivering orders of magnitude computational speedups.

Optimality conditions for sensor placement in OED

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Keywords: Optimal experimental design, sensor placement, convex optimisation, optimality conditions, A-optimality

Within the field of optimal experimental design, sensor placement refers to choosing the most informative sensor locations out of a set of candidate locations. In terms of obtaining optimal reconstruction for inverse problems, this can be formulated as the problem

$$\underset{w\in[0,1]^m,\, \|w\|_0\leq m_0}{argmin} \quad \mathcal{J}(w)$$

where $w = (w_1, \dots, w_m) \in \mathbb{R}^m$ acts as a mask on the measured data, with $w_k = 1$ corresponding to placing the k-th sensor and $w_k = 0$ corresponding to not placing it, and $m_0 \leq m$ is the number of sensors to place. Here, $\mathcal{J}$ measures the quality of the reconstruction of f in the inverse problem

$$\mathrm{Diag}(w)Ff = g, \qquad F : X \to \mathbb{R}^m;$$

especially important are applications where the unknown f is infinite-dimensional and the number of sensor locations $m \in \mathbb{N}$ is large.

These challenges led the development of the results of [1], where we proved a surprising optimality result, showing that even when the problem constraints are relaxed to $\|w\|_1 \leq m_0$, rendering the optimisation problem continuous but non-binary, a large number of indices are *dominant*, i.e. satisfying $w_k = 1$ exactly, or *redundant*, satisfying $w_k = 0$ exactly.

Building on this, we discuss how this result leads to an efficient continuation algorithm for identifying near-optimal binary designs, and how low-rank techniques can be utilised to fully take advantage of this formulation. Lastly, we discuss extensions to passive imaging, where the effect of data correlation and the non-convexity of the objective introduces significant complexity.

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Self-Supervised Sparse-Dense Optical Resolution Photoacoustic Microscopy

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Keywords: Photoacoustic Microscopy; Self-Supervised Learning; Image Reconstruction

We present a data-efficient reconstruction method for high-resolution imaging in optical-resolution photoacoustic microscopy (OR-PAM) [2]. Our approach combines sparse-dense sampling with self-supervised learning. In the first step, image data are acquired by sampling both fully sampled and sparsely sampled regions: fully sampled regions provide local high-frequency information, while sparsely sampled regions capture global image structure at lower acquisition cost. In a second step, a deep neural network is trained in a self-supervised fashion to infer the missing high-resolution information by leveraging the correlation between densely and sparsely sampled regions. We demonstrate that the proposed method yields high-quality reconstructions while significantly reducing the acquisition time. Numerical results in OR-PAM imaging show that our approach performs well at low data rates [1].

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Rethinking Self-Supervised Learning in Inverse Problems for Imaging

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Keywords: Self-supervised learning; task-driven imaging

Self-supervised learning has recently gained popularity in the context of image reconstruction, yet its precise role and definition within inverse problems are not so clearly defined. Unlike classical supervised approaches, self-supervised methods leverage the structure of the data or physics of the problem itself to generate learning signals without relying on paired ground truth data. In this talk, we position the term ”self-supervised” as part of a broader continuum that includes synthetic data generation, task-driven imaging, and model-based learning. These methods do not form mutually exclusive categories but instead represent a spectrum of data and model interactions that can be exploited to advance imaging pipelines.

We present a modular view on inverse problems in imaging that explicitly decouples the imaging process into interpretable and learnable components. This approach encompasses synthetic data generation tailored to specific imaging geometries, learning-based models for image reconstruction and parameter estimation within the image formation process, and superresolution techniques that enhance spatial resolution. Additionally, we incorporate task-driven imaging strategies, where reconstruction is informed by downstream analysis objectives. Integrating these components enables adaptable and generalizable solutions across a wide range of inverse problems, as demonstrated through experimental results in various imaging applications.

Learning from limited data: a Self-Supervised DEQ approach for Sparse-Angle CT Reconstruction

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Keywords: Deep equilibrium models; Tomographic imaging; Self-Supervised

Most Deep Learning approaches for solving imaging inverse problems require a significant number of paired training data. In medical applications, such as computed tomography (CT), acquiring ground truth images can be difficult. For this reason, self-supervised methods emerged as an alternative to exploit the acquired measurements. In this talk we present a self-supervised Deep Equilibrium (DEQ) framework for sparse-angle CT reconstruction, whose training is performed using the undersampled measurements. The proposed model, named TomoSelfDEQ [1], is a generalization of SelfDEQ [2] to non-unitary operators. In particular, we establish theoretical guarantees showing that, under suitable assumptions, the self-supervised updates match those of fully-supervised training with a slightly modified loss, which includes the forward operator (in our setting the CT forward map). Moreover, numerical experiments confirm the theoretical finding, showing that TomoSelfDEQ achieves state-of-the-art results also in scenarios with a very limited number of angles.

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Levenberg–Marquardt with singular scaling and applications

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Keywords: Levenberg–Marquardt; Semi-norm regularizer; convergence analysis; parameter identification

Inspired by certain regularization techniques for linear inverse problems [1, 2, 3], we study convergence properties of the Levenberg-Marquardt method using singular scaling matrices which corresponds to use a semi-norm regularizer [4]. Under a completeness condition, it is possible to show that the method is well-defined and under an error bound assumption we can establish its local quadratic convergence in the case of zero residual (no noise). For a line-search version of the method, we prove that limit points of the sequence generated by the method are stationary for the sum-of-squares function. We assess the practical performance of the method with some examples of parameter identification in heat conduction problems for which specific singular scaling matrices can be used to improve the quality of approximate solutions with respect to the classical method.

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Global Concave Optimization for the Calderón Problem with Finitely Many Unknowns

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Keywords: Calderón Problem; Global Optimization; Concave Optimization; Convex Machine Learning

Electrical impedance tomography (EIT) tries to recover the conductivity of a body from current/voltage pairs, which are applied and measured on the boundary. The mathematical formulation, i.e., recovering the conductivity map from the associated Neumann-to-Dirichlet operator, yields the famous Calderón problem. Practitioners usually employ some kind of data fitting approach, however these methods are typically only locally convergent and hence unreliable. Following [2], we restate the Calderón problem with finitely many unknowns as a convex semidefinite optimization problem with a concave objective, i.e.

$$\min\ f(x) \quad \text{s.t.} \quad x \in \mathcal{F} \subset \mathbb{R}^n.$$

with a concave function $f : \mathbb{R}^n \to \mathbb{R}$ and a compact, convex set $\mathcal{F}$. For problems of this form globally convergent algorithms based on so called Outer Approximation (OA) are known [1]. We employ a basic OA and a combined Branch-and-Bound OA algorithm to reliably recover conductivities in low dimensions. To make the problem computationally tractable at higher resolutions, we restrict the input space to the submanifold of synthetic lung images and learn a convex parametrization using a suitable autoencoder architecture.

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How to solve inverse elliptic coefficient problems by semidefinite convex optimisation

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Keywords: Inverse problems, shunt electrode model, global convergence

In this talk, we present a novel approach to an inverse coefficient problem in an elliptic partial differential equation, where the goal is to reconstruct a piecewise constant coefficient function from boundary measurements. The unknown is discretized over a fixed pixel partition, yielding a finite-dimensional Calderón-type problem.

We reformulate this into an equivalent optimization problem with a linear objective and a convex matrix inequality constraint. This allows us to use semidefinite programming techniques. Unlike standard approaches, which are locally convergent and sensitive to initialization, our formulation provides global convergence guarantees and includes explicit inverse Lipschitz stability estimates.

By rewriting the convex constraints in a tractable form and computing all constants needed for algorithm design, we arrive at a practically usable method. Numerical experiments confirm both the stability and global convergence of the approach, even at high resolutions.

Qualitative acoustic imaging of structures buried in a layered background medium

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Keywords: acoustic imaging; inverse scattering; qualitative imaging; ultrasound imaging

We devise an inversion algorithm to tackle the problem of imaging structures embedded in a multi-layered background medium when multistatic acoustic data is available, in particular a scheme is devised to work under ultrasound experimental real data measured at a non-destructive testing (NDT) facility.

The proposed imaging algorithm is a variation of the class of qualitative inverse scattering methods, and similarly shows some attractive features: multiple scattering effects are completely accounted for, the implemented imaging algorithm is fast, no forward solver is required, and the number of hidden objects is not known a priori.

Specifically the experimental setup that we consider is an array of emitters and receivers consisting of 32 acoustic transducers located away from a region of interest with objects embedded in a known background like water. We consider ultrasound frequency ranges of a few MHz, and the scattering size of the hidden objects are neither small or large so that multiple scattering should not be disregarded.

We present results that show that the ultrasound imaging algorithm is robust for real data measured at the NDT laboratory, with virtual independence on the physical properties or on the number of scatterers.

Partial data inverse problem for the semi-linear wave equation

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Keywords: semi-linear wave equation; time-dependent; partial data

In this talk we discuss a partial data inverse problem for the semi-linear wave equations on a Riemannian manifold with boundary conditions, and thus generalizes the main result in [2] to a semi-linear wave equation with time-dependent coefficients. Furthermore, unlike [3], which considered the recovery of time-dependent coefficients of linear wave equations, we do not impose size restrictions on the measurement set. Lastly, we address the inverse problem in $\mathbb{R}^n$ with Euclidean metric by adapting the argument from [1].

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Determining the time dependent convection term and matrix valued potential in a heat equation from partial data

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Keywords: Inverse problem, heat equation, time-dependent coefficient

We establish the unique recovery of the time-dependent convection term and the matrix valued potential appearing in the heat equation, using measurements on a possibly small subset of the boundary. Specifically, we show that, up to a gauge, the convection term and the density coefficient can be recovered from the Dirichlet-to-Neumann map measured on a small open subset of the boundary.

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An inverse problem for a time-dependent convection-diffusion equation

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Keywords: Convection-diffusion equation; Partial boundary data; Time-dependent coefficient

In this talk, we discuss partial and local data inverse problems for the time-dependent convection-diffusion equation in a bounded domain. For the partial data setting, we show that the time-dependent convection and density terms can be uniquely recovered up to the natural gauge invariance using the knowledge of the Dirichlet to Neumann map measured on a small open subset of the boundary. If time permits, we will also discuss the local data problem, where part of the boundary is treated to be inaccessible, upon assuming the inaccessible part is flat, we establish uniqueness in recovering the same coefficients from the measurements taken outside the inaccessible part of the boundary. The latter problem is joint work with Pranav Kumar.

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Momentum ray transforms - inversion and applications to inverse problems

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Keywords: momentum ray transform; fractional divergent beam transform; higher order Calderón-type inverse problems

Momentum ray transforms are weighted transforms that integrates symmetric tensor fields of rank m over lines in Euclidean space with weights that are powers of the integration parameter. This is a generalization of the standard longitudinal ray transforms which has attracted significant attention due to its several tomographic applications. We first consider the inversion of these transforms as well as the inversion of the normal operators associated to these transforms. Next we consider applications of these transforms in the resolution of higher order Calderón-type inverse problems with tensor perturbations. In the final part of the talk, we consider fractional versions of momentum ray transforms and derive explicit reconstruction formulas and stability results for vector fields and symmetric 2-tensor fields given fractional momentum ray transform data. This talk is based on joint works with Sombuddha Bhattacharyya, Shubham Jathar, Manas Kar, Ramesh Manna, Rahul Raju Pattar, Suman Kumar Sahoo and Vladimir Sharafutdinov.

Shape optimization of regions supporting boundary conditions.

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Keywords: Asymptotic analysis, shape derivative, topological derivative

We consider shape and topology optimization problems in a domain Ω, in which a region $G \subset \partial\Omega$, where specific boundary conditions are applied, is part of the design variables. The numerical method we propose combines :

- an adapted notion of Hadamard shape derivative to evaluate the sensivity of the cost function with respect to 'small' perturbations of the boundary of G within $\partial\Omega$,
- asymptotic analysis to compute the sensitivity of the cost function with respect to the nucleation of a new connected component to G, in the form of a 'small' surface disk.

We give 3d numerical examples in acoustics and structural mechanics, to illustrate the salient features of the method [1].

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Broadband approximate cloaking: feasibility vs. infeasibility

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Keywords: Helmholtz equation; Drude-Lorentz model; Approximate cloaking

In the first part of the talk, I will present our recent work on the feasibility of broadband approximate cloaking in the context of the Helmholtz equation. We employ a transformation optics-based cloaking scheme in which the refractive index incorporates a DrudeLorentz model. By appropriately tuning the material parameters such as sending the resonant frequency of the DrudeLorentz term to infinity it is possible to achieve any desired level of approximate cloaking (in the far field), uniformly over any finite frequency band. In the second part of the talk, I will discuss some known infeasibility results reported in the literature, along with the mathematical techniques used to establish them.

Ray transform of symmetric tensor fields on 2-dimensional Riemannian manifolds with conjugate points

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Keywords: geodesic ray transform; conjugate points; cancellation of singularities

We study the microlocal properties of the geodesic ray transform of symmetric m-tensor fields on 2-dimensional Riemannian manifolds with boundary allowing the possibility of conjugate points. As is known from an earlier work on the geodesic ray transform of functions in the presence of conjugate points, the normal operator can be decomposed into a sum of a pseudodifferential operator and a finite number of Fourier integral operators under the assumption of no singular conjugate pairs along geodesics, which always holds in 2-dimensions. We use the method of stationary phase to explicitly compute the principal symbol of the pseudodifferential operator and each of the Fourier integral operator components of the normal operator acting on symmetric m-tensor fields. Next, we construct a parametrix recovering the solenoidal component of the tensor fields modulo Fourier integral operators, and prove a cancellation of singularities result, similar to an earlier result of Monard, Stefanov and Uhlmann for the case of geodesic ray transform of functions in 2-dimensions.

Discrete microlocal methods

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Keywords: microlocal/semiclassical analysis; sampling; wavefront set; discrete approximation

The wavefront set and Fourier integral operators are among a collection of harmonic and microlocal analytic methods that have widespread applications in inverse problems such as imaging and dynamics. While these tools have significantly driven the theory, there is a gap between the intrinsic asymptotic nature of semiclassical/microlocal analysis on cotangent bundles and the low resolution regime of finite sampling at discrete points. We introduce methods for quantitatively estimating FIOs and the microlocal content of tempered distributions on the continuous side with corresponding matrix-vector dynamics from finite samples on the discrete side. In particular, we will see how to characterize, with finite and non-asymptotic estimates, the relationship between the number of samples and the quality/rate of the resolution of propagations of singularities and wavefront sets. Turning this the other away, we will briefly discuss by example, how some existing numerical methods can be seen through the lens of discrete microlocal analysis and thereby, how these techinques can give rigorous estimates on the quality of such approximation schema (that may otherwise remain heuristic). Time permitting, we will indicate rates of spectral resolutions of FIOs from sampled matrix approximations.

X-ray transforms and transport twistor spaces

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Keywords: geodesic X-ray transform; transport twistor space

Some problems of integral geometry are concerned with the recovery of functions and tensor fields from their integrals along families of curves on a manifold. On Riemannian surfaces, where the curves considered are geodesics, the resolution of these problems has passed through important notions (so-called "fiberwise holomorphicity") which were hinting at an underlying complex-geometric picture, although this link was only clarified a few years ago by Jan Bohr and Gabriel Paternain through the notion of transport twistor space.

Given (M, g) a Riemannian surface with unit tangent bundle SM, the transport twistor space is a complex surface containing SM as a boundary component, and whose complex structure degenerates along the geodesic flow. Such a construction has allowed to build correspondences between "fiberwise holomorphic" geodesically invariant distributions on SM, and genuine holomorphic functions on twistor space. This viewpoint has already served as a helpful conceptual tool to reframe and approach geometric inverse problems questions, and leaves many open questions regarding the properties, classification, and rigidity of transport twistor spaces, and their further use in inverse problems.

In this talk, I will report on recent joint works with Bohr and Paternain, one on the existence of "good" coordinates on these twistor spaces (namely, maps with "holomorphic blowdown structure"), see [1], and the other addressing (biholomorphism) rigidity issues of transport twistor spaces, see [2], as well as some implications of these results.

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Guillarmou's Π operator for general flows and applications to the magnetic ray transform

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Keywords: Normal operator, magnetic ray transform, microlocal analysis, Anosov flow, stability estimates

Let X be the generator of a transitive Anosov flow on a closed manifold M. Using the theory of anisotropic Sobolev spaces from [2], it can be shown that the resolvents $R_\pm(z) := (\mp X - z)^{-1}$ have the following Laurent expansions in a neighborhood of $z = 0$:

$$R_\pm(z) = R_\pm^{\mathrm{hol}}(0) - \frac{\Pi_0^\pm}{z} + O(z).$$

Guillarmou defined the operators

$$\Pi := -(R_+^{\mathrm{hol}}(0) + R_-^{\mathrm{hol}}(0)), \quad \Pi_m = \pi_{m*}(\Pi + 1 \otimes 1)\pi_m^*,$$

and showed that Π_m is the analog of the normal operator for the X-ray transform on closed Anosov Riemannian manifolds [4]. Here $\pi_m^*\colon C^\infty(S^m T^* M) \to C^\infty(SM)$ is the usual pullback from tensors to functions, and π_{m*} is its adjoint. In our work, we extend this analysis to more general flows, for example, we show that $\Pi_0 \in \Psi_{\mathrm{cl}}^{-1}(M)$. In the particular case of a magnetic Anosov flow, based on the potential-solenoidal decomposition from [1], [5] we show that $\Pi_m \in \Psi_{\mathrm{cl}}^{-1}(M)$. Finally, using the Approximate Livšic Theorem from [3], we prove an stability estimate for the magnetic ray transform.

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Electrical Impedance Tomography in 2025

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Keywords: Electrical Impedance Tomography; Reconstruction Algorithms; Pulmonary Imaging

Electrical impedance tomography (EIT) is a medical imaging technique in which low frequency, low amplitude current is applied on electrodes, the resulting voltage is measured, and an inverse problem is solved to reconstruct the admittivity distribution in the interior of the body. Direct reconstruction methods based on inverse scattering and D-bar equations have the advantage of being fast, nonlinear, and without concerns of convergence to local minima. Iterative methods have the advantage of being fast and easy to implement, especially in 3-D. The field of EIT has grown tremendously in the past 45 years since Calderón's seminal paper, presented in 1980 in Rio de Janeiro. This talk will present some state-of-the-art reconstruction approaches from a clinical perspective and current challenges in the field of interest to mathematicians. Images in 2-D and 3-D computed from data collected on patients at Children's Hospital Colorado will be shown.

On the regularization property of Levenberg-Marquardt with Singular Scaling for inverse problems

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Keywords: Inverse problems; Levenberg-Marquardt; Singular Scaling Matrix; Regularization

Recently, in [1], a Levenberg-Marquardt method (LMM) with Singular Scaling was analyzed and successfully applied in parameter estimation problems in heat conduction where the use of a particular singular scaling matrix (semi-norm regularizer) provided approximate solutions of better quality than those of the classic LMM. Here we propose a regularization framework for the LevenbergMarquardt method with Singular Scaling (LMMSS) applied to nonlinear inverse problems with noisy data. Assuming that the noise-free problem admits exact solutions (zero-residual case), we consider the LMMSS iteration where the regularization effect is induced by the choice of a possibly singular scaling matrix and an implicit control of the regularization parameter. The discrepancy principle is used to define a stopping index that ensures stability of the computed solutions with respect to data perturbations. Under a new Tangent Cone Condition, we prove that the iterates obtained with noisy data converge to a solution of the unperturbed problem as the noise level tends to zero. This work represents a first step toward the analysis of regularizing properties of the LMMSS method and extends previous results in the literature on regularizing LM-type methods.

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Leveberg-Marquardt methods for systems of equations with additional convex constraints

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We consider the application of Levenberg-Marquardt method for solving systems of nonlinear equations under additional convex constraints, i.e. the solution must belong to a closed and convex set C. Several possibilities arise. One consists of adding the convex constraints to the subproblems. Under this option, superlinear convergence is achieved under a judicious choice of the regularization parameters, but the subproblems become rather hard (minimization of a quadratic objective subject to general convex constraints, instead of solving just a linear system). A second option consists of taking a Levenberg-Marquardt step, ignoring the convex constraints, and then projecting the resulting point onto C. A third option consists of projecting such resulting point onto a hyperplane separating it from C. These two options have easier iterations (particularly the third one: in each iteration a linear system is solved and then a closed formula produces the projection onto the hyperplane), but convergence is at most linear, due to the projections. We propose a fourth option, for the case in which C is of the form $C = \{x \in \mathbb{R}^n : g_i(x) \leq 0 \ (1 \leq i \leq m)\}$ with convex g_i's. The subproblems of the method incorporate second order information on the g_i's to the quadratic objective and linear approximations of the g_i's as constraints, resulting in quadratic programming subproblems, reasonably easy to solve. We conjecture that the method enjoys superlinear convergence.

The training of SVMs through a smooth sparse-promoting-regularized squared hinge loss minimization

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Keywords: SVM:s; Binary classification

Support Vector Machines (SVMs) perform binary classification tasks, under a supervised framework. They often outperform other supervised methods and are one of the most popular approaches in the field of machine learning. We investigate the training of SVMs through a smooth sparse-promoting-regularized squared hinge loss minimization. This choice paves the way to the application of quick training methods built on majorization-minimization approaches, benefiting from the Lipschitz property of the loss function. In addition to that, the method allows us to handle sparsity-preserving regularizers, which improves performance by promoting the selection of the most relevant features. Numerical tests and comparisons conducted on three different datasets demonstrate the good performance of the proposed methodology in terms of qualitative metrics (accuracy, precision, recall, and F1 score) and computational cost [1].

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Tomogram alignment without reconstruction

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Keywords: tomogram; alignment; consistency

We present an explicit formula for the best approximation of a real sinogram in the context of parallel-beam tomography, derived by imposing generalized Ludwig-Helgason conditions through arbitrary ridgelets. This formula has multiple applications, including the removal of artifacts, such as ring artifacts commonly observed in absorption tomography, particularly in synchrotron-based studies. By obtaining the best approximation, we propose a model for recovering the optimal sinogram using a physics-inspired artificial intelligence approach. The effectiveness and robustness of the method are demonstrated through experiments with synthetic and real data. The problem of tomographic transmission using x-rays generated at synchrotron facilities, specifically in the context of parallel-beam geometry. The Radon transform serves as the fundamental mathematical framework for modeling the associated inverse problem. A typical tomographic measurement provides a three-dimensional dataset, denoted as $\mathbf{Y}$, where each sliced plane corresponds to a radiographic $N \times N$ image of the sample for a constant projection angle θ. The angle θ varies over a discrete mesh with V points spanning the interval $[0, \pi]$. The dimensions (N, V) are characteristic of an imaging beamline, which typically operates with $N \sim 2048$ and $V > 2048$. Consequently, $\mathbf{Y}$ represents a large dataset. In the case of parallel-beam tomography, data can be processed slice-by-slice, commonly referred to as sinograms. Within the standard data-processing workflow, the alignment of the sinogram block $\mathbf{Y}$ is a critical step; however, this article assumes that $\mathbf{Y}$ is already pre-aligned. Despite this alignment, the sinograms often exhibit imperfections, deviating from the expected forward model defined by the Radon transform.

In this work, an optimization algorithm is employed to compute the distance between the measured sinogram y and an ideal sinogram g. An ideal sinogram g satisfies a set of consistency conditions [**?**], denoted by $\mathcal{S}$. If $\mathcal{S}$ is convex, calculating this distance is equivalent to projecting y onto a convex set. A similar approach was previously explored before, where $\mathcal{S}$ was defined as a specific set based on five consistency conditions. This method was used to obtain the best approximation of the sinogram prior to applying convolution-backprojection algorithms. Their approach is highly effective as it relies on the Chebyshev-Fourier transform of the measured sinogram. Additionally, the consistency set $\mathcal{S}$ was carefully selected to ensure that each projection could be computed analytically. The approach described here differs from the literature in both the definition of the consistency set $\mathcal{S}$ and the algorithm used to compute the optimal distance point. The projection onto our consistency set is derived as a fixed point of the optimality conditions for a penalized objective function. Under certain conditions, the sequence g_k will converge to the projection of y as long as the penalty sequence satisfies $r_k \to 0$. We propose a definition of $\mathcal{S}$ based on the Helgason-Ludwig conditions, but designed to be more practical for computational purposes. In real measurements, several experimental conditions can affect y, and consequently g. Examples of these conditions include exposure time, number of projections, alignment, detector distance, and whether the beam is monochromatic or polychromatic, among others. The objective of this paper is to provide a fast algorithm to quantify the distance between a given measurement and an ideal dataset. This algorithm can be seamlessly integrated into the control systems of a beamline or other tomographic scanning devices.

Analysis of generalized iteratively regularized Landweber iterations driven by data

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Keywords: data-driven regularization; Landweber method

We introduce and analyze novel data-driven variants of the Iteratively Regularized Landweber Method for solving linear and nonlinear ill-posed inverse problems [1, 2]. Our approach leverages both labeled and unlabeled data to enhance the regularization scheme. In the presence of labeled data, we propose a learning-based modification in which training samples are used to estimate the behavior of a black-box operator that guides the iteration process. For unlabeled data, we replace the classical damping term with either the average or the geometric mean of the data, following a fully data-driven perspective. We establish convergence and stability results in infinite-dimensional Hilbert spaces and validate our findings through numerical experiments, particularly for inverse problems involving the Radon transform.

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Multiresolution low-rank regularization in dynamic tomography

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Keywords: Low-rank regularization; Wavelets; Dynamic tomography

MultiResolution Low-Rank decomposition is introduced for regularization of dynamic image sequences. The decomposition applies a local low-rank decomposition on a time sequence of coefficients from spatially applied wavelet transforms. Its use as a regularization functional is illustrated and numerically tested for dynamic X-ray tomography in comparison to other low-rank methods. The results suggest it is similar to traditional locally low-rank decomposition but produces less severe block artifacts.

All codes are available on Github: `https://github.com/tommheik/WaveletLowRank`.

Handling regularization and discretization in the reconstruction for elliptic inverse problems

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Keywords: Regularization; Discretization; Total variation

We consider the reconstruction issue for elliptic inverse problems such as the electrical impedance tomography or the determination of inhomogeneities by scattering measurements. Due to their severe ill-posedness, a regularization technique is needed. Moreover, the discretization used for the numerical implementation may also be an important cause of instability. We investigate how to handle simultaneously the regularization term and the discretization so that the solution to the corresponding regularized and discrete inverse problem is a good approximation of the solution to the original one.

The inverse Born series for the reconstruction of Kerr nonlinearities.

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Keywords: inverse scattering; Kerr effect, nonlinear wave equation

We investigate the Born and inverse Born series for a scalar wave equation with linear and nonlinear terms, the nonlinearity being cubic of Kerr type. We show conditions which guarantee convergence of the inverse Born series, enabling recovery of the coefficients of the linear and nonlinear terms. Furthermore, we show that if the unknown perturbation is only in the nonlinear term, an arbitrarily strong nonlinearity can be reconstructed for sufficiently small data. Similar convergence results hold for general polynomial nonlinearities. Our results are illustrated with numerical examples.

Reconstruction for the Calderón problem with Lipschitz conductivities

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Keywords: Calderón problem, reconstruction, low regularity

The Calderón problem seeks to determine the conductivity of the interior of a body from electrical measurements on its boundary. In the eighties, a reconstruction procedure was provided for twice continuously differentiable conductivities. In this talk, we will address the reconstruction in the case that the conductivities are only Lipschitz continuous. For that, we will introduce Sobolev spaces with norms depending on auxiliary parameters, in which we can construct suitable CGO solutions.

Lagrangian approach and shape gradient for inverse problem of breaking line identification in solid

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Keywords: Variational inequality; Shape optimization; Directional semidifferentiability

A class of inverse identification problems constrained by variational inequalities is studied with respect to its shape differentiability. The specific problem appearing in fracture mechanics describes elastic bodies with a breaking line subject to contact conditions between its faces. Based on the Lagrange multiplier approach and smooth Lavrentiev penalization, a semi-analytic formula for the shape gradient of the Lagrangian linearized on the solution is proved, which contains both primal and adjoint states. It is used for the descent direction in a gradient algorithm for identification of an optimal shape of the breaking line from boundary measurements. The theoretical result is supported by numerical simulation tests of destructive testing in 2D configuration comparing the problems with and without contact.

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Ill-Posed Problem for the Black-Scholes Equation Solution and Machine Learning

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Keywords: Black-Scholes equation, European call options, geometric Brownian motion, probability theory, ill-posed problem, quasi-reversibility method, Carleman estimate, trading strategy.

In the previous paper (*Inverse Problems, 32, 015010, 2016*), a new heuristic mathematical model was proposed for accurate forecasting of prices of stock options for 1-2 trading days ahead of the present one. This new technique uses the Black-Scholes equation supplied by new intervals for the underlying stock and new initial and boundary conditions for option prices. The Black-Scholes equation was solved in the positive direction of the time variable. This ill-posed initial boundary value problem was solved by the so-called Quasi-Reversibility Method (QRM). This approach, with an added trading strategy, was tested on the market data for 368 stock options and demonstrated good forecasting results.

We use the geometric Brownian motion to provide an explanation of that effectiveness using computationally simulated data for European call options. We also provide a convergence analysis for QRM. The key tool of that analysis is a Carleman estimate. To enhance these results, Neural Network Machine Learning is applied at the second stage. Real market data are used. Results of the Quasi-Reversibility Method and the Machine Learning method are compared in terms of accuracy, precision, and recall.

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Path integrals, inverse problems, and functorial field theory

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Keywords: path integrals, functorial field theory

Path integrals originated as a way to describe the behavior of physical systems in quantum mechanics and quantum field theory using integrals over infinite-dimensional spaces of physical fields. Recently, the underlying mathematical machinery became employed in a wide range of mathematical subjects, including inverse problems, stochastic analysis, and many others. Formalizing path integrals is difficult and cannot use the traditional machinery of measure theory except in the easiest cases of Gaussian measures. In this talk, I will discuss functorial field theory, an approach to path integrals based on axiomatizing their properties such as the generalized Fubini theorem. I will start by reviewing my recent work with Dan Grady on the geometric cobordism hypothesis and locality of fully extended nontopological functorial field theories. I will then apply these results to explicitly compute, in terms of homotopy coherent representations of Lie groups, the space of 2-dimensional fully extended conformal field theories.

From inverse coefficient problems to semidefinite optimization

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Keywords: Inverse coefficient problem, semidefinite optimization, global convergence

Several applications in medical imaging and non-destructive material testing lead to inverse elliptic coefficient problems, where an unknown coefficient function in an elliptic PDE is to be determined from partial knowledge of its solutions. This is usually a highly non-linear ill-posed inverse problem, for which unique reconstructability results, stability estimates and global convergence of numerical methods are very hard to achieve.

In this talk we will consider the inverse coefficient problem of electrical impedance tomography (EIT) with finitely many measurements and a finite desired resolution. We will discuss new results for writing this non-linear inverse problem as a convex or concave semidefinite optimization problem that allow globally convergent reconsruction algorithms.

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An efficient pseudodifferential preconditioner for the Helmholtz equation

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Keywords: Wave propagation; high frequency acoustics; log-linear complexity

We formulate a new pseudodifferential preconditioner for the Helmholtz equation in variable wave speed and absorption. The pseudodifferential operator is associated with an approximate inverse to the symbol of the Helmholtz operator. We propose a fast evaluation of the preconditioner by interpolating its symbol (not as a function of the high-dimensional phase-space variables [1, 2]), but as a function of the wave speed itself. Since the wave speed is a real-valued function, this approach renders a *univariate* interpolation even when the original problem is posed in a multidimensional physical space. As a result, the number of interpolation points is small, and the interpolation coefficients can be computed using the fast Fourier transform. The overall computational complexity is log-linear with respect to the degrees of freedom as inherited from the fast Fourier transform.

In this talk, we present results from numerical experiments to illustrate the effectiveness of the preconditioner to solve the discrete Helmholtz equation using the GMRES iterative method. We also show the implementation of an absorbing layer for scattering problems using a complex-valued wave speed. Limitations and possible extensions will be discussed. More details are found in a recent publication [3].

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Analysis of Neural Activation in Time-dependent Membrane Capacitance Models

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Keywords: FitzHugh-Nagumo equation; Action potential; Neurostimulation

We will present a modified formulation of a neuron model, in which we include a time-dependent membrane capacitance. The goal is, for this model, to understand and control the generation of action potentials for certain capacitance dynamics. We show that we need large enough and abrupt variations in the capacitance to generate action potentials; we provide explicitly constructed simple capacitance profiles that generate action potentials; and we also show that high frequency chances in the capacitance may not result in the generation of action potentials. The theoretical results that we obtain can have implications for the design of ultrasound-based or other neuromodulation strategies acting in the membrane capacitance of neurons.

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Robin perturbation of the boundary condition of an elliptic PDE.

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Keywords: Asymptotic analysis, integral representation, boundary impedance

Asymptotic expansions of the solution to an elliptic PDE in the presence of inclusions of small size have found succesfull applications in inverse problems, in particular for the detection of inhomogeneities. In this talk, we consider situations where the perturbations are not caused by internal inhomogeneities, but take place on the boundary. Building up on our previous work [1], we assume that a homogeneous Dirichlet or Neumann boundary condition is replaced by a Robin condition on a small subset ω_ε of the domain boundary. We characterize the first term in the asymptotic expansion of the solution, in terms of the relevant measure of smallness of ω_ε, and study how the convergence of the expansion depends on the impedance of the Robin boundary condition.

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The inverse (poro)elasticity problem

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Keywords: Inverse elasticity, Large deformations, Computational mechanics

The inverse elasticity problem can be simply stated as: given a deformed configuration and the forces that act on it, find an initial stress-free configuration such that when the given forces are applied to it, one recovers the given deformed configuration. Surprisingly, this problem can be framed as a (direct) elasticity one, whose mathematical properties are inherited from the original direct problem if the underlying material is sufficiently regular. In this talk, I will review this problem and its main mathematical properties. After this brief introduction, I will show some artifacts that appear when solving this problem, such as self-intersections and geometrically incompatible solutions. The talk will finish with an extension of this system to poroelastic materials, where I will show that the strong form of the equations does not allow for a weak formulation, and this requires some special treatment. All models will be shown to work in realistic heart geometries.

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Manifold Denoising using Convex Relaxations

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Keywords: Manifold Denoising; Cryo-Electron Microscopy

We investigate the problem of manifold denoising, where observations are noisy samples drawn from a smooth manifold embedded in a high-dimensional Euclidean space. Consider $\mathcal{M} \subset \mathbb{R}^n$ to be a C^2-smooth submanifold of dimension d. Let $\mu \in \mathcal{M}$ denote a point sampled from the uniform distribution on $\mathcal{M}$. Let $z \in \mathbb{R}^n$ denote a random vector sampled from the Gaussian distribution in $\mathbb{R}^n$ having mean zero and covariance $\sigma^2 I_n$ that is independent of μ. Let $y = \mu + z$ be observed. Let $\mathcal{K} = \mathrm{conv}(\mathcal{M})$ denote the convex hull of $\mathcal{M}$. Let $\mathcal{P}_{\mathcal{K}}$ denote the projection operator from $\mathbb{R}^n$ to $\mathcal{K}$ and the projection of y on the convex hull $\mathcal{K}$ be given by $\hat{\mu} = \mathcal{P}_{\mathcal{K}}(y)$. By leveraging results in empirical process theory, we derive upper bounds for $\|\mu - \hat{\mu}\|_2$ that depend explicitly on quantities associated with the manifold, such as its reach, diameter, and volume. Additionally, we apply our results to denoising images coming from Cryo-Electron Microscopy.

Interpretable and interactive learned regularizers

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Keywords: Learned regularizers; Bayes Hilbert space; Concept whitening

Learned regularizers—interpretable as Bayesian priors—achieve state-of-the-art performance on many inverse problems, particularly in medical and seismic imaging. However, because these regularizers are neural networks trained from data, they often behave as "black boxes." Our research aims to develop learned regularizers that are interpretable not only mathematically, but also in terms of concepts from the end-users application domain. To this end, we adapt a strategy known as *concept whitening*, a technique that encourages neural networks to route all information from a given set of auxiliary concept training sets through a prescribed subset of neurons. By inspecting the activations of these neurons on a new input, users can infer how each concept contributes to the networks output. In this talk, we reinterpret concept whitening as an orthogonalization procedure for probability measures in a Bayes Hilbert space. This perspective provides new mathematical foundations and motivates novel computational strategies. Ultimately, our goal is to design interactive regularizerstools that allow users to adjust Bayesian priors and explore the space of possible reconstructions in a mathematically principled and interpretable way.

The learned range test method for the inverse inclusion problem

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Keywords: Inverse inclusion problem, electrical impedance tomography, domain sampling method, machine learning, neural networks

We consider the inverse problem consisting of the reconstruction of an inclusion B contained in a bounded domain $\Omega \subset \mathbb{R}^d$ from a single pair of Cauchy data $(u|_{\partial\Omega}, \partial_\nu u|_{\partial\Omega})$, where $\Delta u = 0$ in $\Omega \setminus \overline{B}$ and $u = 0$ on ∂B. We show that the reconstruction algorithm based on the range test, a domain sampling method, can be written as a neural network with a specific architecture. We propose to learn the weights of this network in the framework of supervised learning, and to combine it with a pre-trained classifier, with the purpose of distinguishing the inclusions based on their distance from the boundary. The numerical simulations show that this learned range test method provides accurate and stable reconstructions of polygonal inclusions. Furthermore, the results are superior to those obtained with the standard range test method (without learning) and with an end-to-end deep neural network, a purely data-driven method.

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Quantum algorithms for inverse problems

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Keywords: Quantum computing; Grover's algorithm

Quantum computing offers new possibilities in the study of inverse problems. In this talk we will establish a general framework for the study of discrete inverse problems by means of quantum computing, highlighting the main principles involved and giving examples of applications. The talk is self-contained, in the sense that no previous knowledge of quantum computing is required. This is part of a joint work with professors Matti Lassas and Maarten de Hoop.

The elastic ray transform

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Keywords: Elasticity; Integral geometry

In this talk, we introduce and study the elastic ray transform. An elastic tensor field is contracted against polarization vectors and integrated over lines in a Euclidean space, giving the elastic ray transform data. We prove a kernel characterization for the ray transform in terms of two differential operators. The proof is based on the Fourier slice theorem and a certain Helmholtz-type decomposition for elastic tensor fields. The study of this transform is motivated by the linearization of the travel time problem in elasticity. This talk is based on joint work with Joonas Ilmavirta and Teemu Saksala.

The backscattering problem for time-dependent potentials

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Keywords: scattering theory; time dependent potentials

We study the inverse backscattering problem for time-dependent potentials. We prove uniqueness and Lipshitz stability for the recovery of small potentials.

Inverse spectral problems for collapsing manifolds

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Keywords: Gel'fand's inverse problem; collapsing manifolds

Gel'fand's inverse spectral problem concerns the determination of a Riemannian manifold from the Laplacian spectral data $(\lambda_j, \phi_j|_B)$ measured on B where B is either the boundary or an open subset of the manifold. We review recent progress on the stability of the problem. Next, we consider the unique solvability and stability of the inverse spectral problem for Riemannian orbifolds, that is, spaces which are locally isometric to the quotient of Riemannian manifolds by finite group action. This talk is based on joint works with Y. Kurylev, M. Lassas and T. Yamaguchi.

An Inverse Problem for Hyperbolic PDEs on Complete Riemannian Manifolds

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Keywords: Analysis of PDEs; Spectral Theory

Let (N, g) be a complete Riemannian manifold (potentially not compact). If $\mathcal{L} : C^\infty(N) \to C^\infty(N)$ is a symmetric operator known as the Magnetic-Schrödinger operator, then we can consider the hyperbolic problem

$$\begin{cases} (\partial_t^2 + \mathcal{L})u^f(t,x) = f(t,x), & \text{for } (t,x) \text{ in } (0,\infty) \times N \\ u^f(0,x) = \partial_t u^f(0,x) = 0, & \text{for } x \text{ in } N. \end{cases}$$

If $\mathcal{X} \subset N$ is open set then we can define the local source-to-solution operator $\Lambda_{\mathcal{X}}$ to be the map for $f \in C_0^\infty((0,\infty) \times \mathcal{X})$ to the solution u^f restricted to $(0,\infty) \times \mathcal{X}$. That is

$$\Lambda_{\mathcal{X}}(f) = u^f|_{(0,\infty)\times\mathcal{X}}.$$

We show that $\Lambda_{\mathcal{X}}$ determines the topological, smooth and Riemannian structure of (N, g) as well as the lower order terms of $\mathcal{L}$ up to a natural gauge.

Accelerating non-convex optimization with deep networks for sparse CT reconstruction

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Keywords: Hybrid Methods; Sparse tomography; Non-Convex Optimization; Regularized Inverse Problem; Medical Image Computing

Reconstructing images from undersampled measurements remains a key challenge in the inverse problems community, particularly for medical imaging applications. Non-convex regularizers such as Total p-Variation (TpV) are widely adopted to promote sparsity and preserve fine structures in the reconstruction; however, optimizing these models is computationally demanding and susceptible to local minima.

In this talk, I will present a twofold contribution. First, I will introduce Deep Guess (DG), a novel strategy for accelerating variational methods by using a pretrained neural network to provide informed initializations [1]. Then, I will introduce the incremental TpV (incTpV) algorithm, a model-based iterative reweighted scheme designed to progressively enhance sparsity in the gradient domain by solving a sequence of convex subproblems. Combining these ideas, I propose incDG, a hybrid framework that preserves the interpretability and theoretical grounding of the variational approach while significantly reducing computation time [2].

I will show experimental results on tomographic reconstruction tasks, demonstrating that these hybrid approaches achieve superior reconstruction quality and computational efficiency, even when trained without ground truth data and tested on out-of-distribution samples.

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Electrical impedance tomography and virtual X-rays

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Keywords: Electrical impedance tomography; complex geometric optics solutions; neural networks

Electrical Impedance Tomography (EIT) is a nonlinear PDE-based imaging modality where a patient is probed with harmless electric currents, and the resulting surface voltages are measured. Most promising uses of EIT are in lung imaging and stroke monitoring. EIT image reconstruction is an ill-posed inverse problem, meaning very sensitive to noise in the data and modelling errors. However, one can use complex geometric optics (CGO) solutions and a nonlinear Fourier transform to do robust medical imaging; this is the so-called regularized D-bar method [1]. A connection between EIT and X-ray tomography was found in [2] using microlocal analysis. Fourier transform applied to the spectral parameter of CGO solutions produces virtual X-ray projections, enabling a novel filtered back-projection type nonlinear reconstruction algorithm for EIT. It is remarkable how this new approach decomposes the EIT image reconstruction process in several steps, where all ill-posedness is confined in two linear steps. Therefore, we can separate the nonlinearity and ill-posedness of the fundamental EIT problem. Furthermore, the new decomposition enables targeted machine learning approaches as only one or two (mathematically well-structured) steps in the imaging chain are solved using neural networks.

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Latent Space Techniques for Diffuse Optical Tomography Inverse Problems

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Keywords: Diffuse Optical Tomography; Latent Space; Generative Network

Diffuse Optical Tomography is a non-invasive medical imaging method that employs near-infrared light to estimate the spatial distribution μ_a of optical properties in biological tissues for diagnostic use. However, due to the strong scattering of light within tissues, the reconstruction in DOT is highly ill-posed [1]. The solution to DOT problems has been tackled via model based approaches, addressing the ill–conditioning with regularization techniques, such as the Elastic Net priors or Bregman iterative techniques [3]. We present here two deep learning approaches.

The first approach [2] consists of a pure data–driven method and employs two autoencoders (AEs), the former acting on the acquired signal y_δ and the latter on μ_a. The latent spaces of the two AEs are connected via a bridge network Σ: the reconstruction of new data is given the concatenation of the encoder on the data, of the bridge Σ and of the decoder on μ_a. This procedure shows robustness with respect to noise and, moreover, its inherent structure provides a source of interpretability.

The second approach couples a generative neural network (GNN) with the forward model, modelled by a Graph Neural Network, resulting hence in a hybrid strategy. The main idea is to see GNN as a *learned prior*, for addressing the ill-posedness of the problem. GNN interacts with the forward model, generating the solution of the inverse problem. The numerical tests [4] showed that incorporating a generative prior in solving the inverse problem improves the accuracy of optical coefficient reconstruction.

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Bregman-relaxations of ℓ_0-regularised criteria with general data terms

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Keywords: ℓ_0-relaxation, non-convex optimisation, Bregman relaxations, non-quadratic data terms

Sparse linear models are widely used in fields such as statistics, computer vision, signal/image processing and machine learning. The natural sparsity promoting regulariser one would like to consider is the ℓ_0 pseudo-norm which is discontinuous, non-convex and hard to minimise when combined with classical, possibly non-quadratic data terms. Bregman relaxations (B-Rex) [2] are exact continuous relaxations of such criteria, meaning that, while still non-convex, they let unchanged the set of global minimizers while removing several local minimisers, thus enjoying a better optimisation landscape, see [1] for quadratic fidelities. In this talk I will illustrate these properties on ℓ_0 regularised models combined with + ℓ_2, Kullback-Leibler [3], and logistic regression data-terms, along with tailored algorithms and adaptive strategies for optimal parameter selection relying on regularisation paths and graduated non-convexity [4].

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A Primal-Dual algorithm with Krylov subspace projection for X-ray computed tomography

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Keywords: primal-dual iteration; krylov subspace; computed tomography

In this talk, we explore the possibilities of integrating Krylov subspace methods into a primal-dual algorithm that solves nonsmooth optimization problems. In particular, the Condat-Vu algorithm is considered due to its simple formulation and therefore low per-iteration computational cost. However, its convergence rate makes it less suitable for large-scale imaging problems where the forward and adjoint operators are expensive to compute. Thus, we propose to project the large-scale problem into low-dimensional Krylov subspaces and analyze the numerical properties of the resulting algorithm.

We mainly consider a 3D industrial tomographic image reconstruction problem as a motivating application, cast as a regularized optimization problem with different nonsmooth terms. Finally, we explore the application of the presented algorithm to estimate the hyperparameters of the related variational formulations via a bilevel learning setting.

Differential estimates for fast first-order multilevel nonconvex optimisation

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Keywords: PDE-constrained, bilevel, parameter learning, single-loop

PDE constraints appear in inverse imaging problems as physical models for measurements, for example, in electrical impedance tomography (EIT). Bilevel optimisation, on the other hand, can be used for optimal experimental design and parameter learning. These types of problems have been traditionally very expensive to solve when combined with nonsmooth regularisation, such as total variation. Recently, effective single-loop approaches have been introduced, both in our work, as well as in the machine learning community. In this talk, we discuss a simple gradient estimation formalisation for very general single-loop methods with flexible choice of outer, inner, and adjoint algorithms [1]. This includes, in particular, primal-dual methods for the inner problem, and conventional iterative solvers (Jacobi, GaussSeidel, conjugate gradients) for the adjoint problem and PDE constraints.

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Edge-preserving Randomized Iterative Methods for X-ray Computed Tomography

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Keywords: computed tomography; regularization

We study sparsity-promoting regularization for X-ray CT reconstruction using the Alternating Direction Method of Multipliers (ADMM) and the Variable Projected Augmented Lagrangian (VPAL) methods. To handle large-scale data efficiently, we apply randomized sampling strategies to solve multi-parameter Tikhonov subproblems arising in each iteration. Our focus includes both convex ℓ_1 and non-convex ℓ_1/ℓ_2 regularization models, the latter offering improved edge preservation but posing greater optimization challenges. Numerical experiments highlight trade-offs in accuracy, computational efficiency, and reconstruction quality across different formulations.

Solution of Mismatched Monotone+Lipschitz Inclusion Problems

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Keywords: Adjoint Mismatch; Splitting Algorithms

Adjoint mismatch problems arises when the adjoint of a linear operator is replaced by an approximation, due to computational or physical issues. This occurs in inverse problems, particularly in computed tomography. In this talk we address the convergence of algorithms for solving monotone inclusions in real Hilbert spaces in the presence of adjoint mismatch. In particular, we investigate the case of a mismatched Lipschitzian operator. We propose variants of the algorithms *Forward-Backward-Half-Forward* and *Forward-Half-Reflected-Backward* allowing to cope the mismatch. We establish conditions under the weak convergence to a solution of these variants is guaranteed. Moreover, the proposed algorithms allow each iteration to be implemented with a possibly iteration-dependent approximation to the mismatch operator, thus allowing this operator to be modified in each iteration.

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Convexification Method in Inverse Problems

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Keywords: Global convergence, Carleman estimates, inverse problem of the epidemiology, mean field games, travel time tomography

The convexification numerical method was first introduced by the speaker in 1995 and 1997 [1, 2] with the goal to avoid the phenomenon of local minima and ravines of conventional least squares cost functionals for coefficient inverse problems. While initial works [1, 2] were purely theoretical ones, currently this method is widely applicable for computations of a variety of coefficient inverse problems. Global convergence is rigorously established for each version of this method. The key tool is the apparatus of Carleman estimates.

In this talk, three versions of the convexification method will be presented: for a coefficient inverse problem of the epidemiology [3], for a coefficient inverse problem for the system of Mean Field Games [4, 5] and for the problem of travel time tomography [6, 7]

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Analysis of Forward and Inverse Problems in Time-Dependent Biot and BiotJKD Poroelasticity Models

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Keywords: Biot and Biot-JKD equations; Mixed boundary value problems; Inverse source problem; Existence and uniqueness results

Several forward problems related to the dynamic poroelasticity equations are examined. In the low-frequency regime, described by Biot's equations, we establish the existence and uniqueness of solutions to a mixed boundary value problem [1].

In the high-frequency regime, particularly in the context of the Biot-Johnson-Koplik-Dashen model, we demonstrate the uniqueness of the weak solution to the corresponding mixed problem, along with its continuous dependence on the initial data. These results hold for both finite and infinite time intervals and are valid in arbitrary spatial dimensions [2].

Furthermore, for the Biot equations in the low-frequency approximation, we investigate an inverse source problem. Specifically, we prove the existence and uniqueness of the time-dependent scalar function representing an unknown external body force density [3].

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Mathematical Theory of Flow-Structure Interactions

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Keywords: Flow Structure Interactions, Long time behavior

Flow-structure interactions are ubiquitous in nature and in ev- eryday life. Flow or fluid by interacting with structural elements can lead to oscillations, hence impacting stability or even safety. Thus problems such as attenuation of turbulence or flutter in an oscillating structure [Tacoma bridge], flutter in tall buildings, fluid flows in flexible pipes, in nuclear engineering flows about fuel elements and heat exchanger vanes are just few prime examples of rel- evant applications which place themselves at the frontier of interests in applied mathematics. In this talk we shall describe mathematical models describing the phenomena, They are based on a 3 D linearised Euler Equation around unsta- ble equilibrium coupled to a nonlinear dynamic elasticity on a 2 D manifold. Strong interface coupling between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDEs, dynamical systems, related harmonic analysis and differential geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some new methodology addressing the issues of control and stability of such struc- tures. Part of this talk is based on recent work with D. Bonheur, F.Gazzola and J. Webster :Annales de LInstitute Henri Poincare Analyse, 2022 , work with A.Balakrishna and J. Webster : M 3AS 2024 and also work completed while the author was a member of the MSRI program Mathematical problem in fluid dynamics at the University of California Berkeley (NSF DMS -1928930).

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Transmission imaging using sweeping Pade approximation of pseudodifferential operators

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Keywords: Imaging; Ultrasound, Pseudodifferential Factorization

Nonlinear ultrasound waves play an increasingly important role in both diagnostic and therapeutic medicine. In this work, we address the ultrasound imaging problem governed by the Helmholtz equation with variable wave speed and fractional attenuation. To tackle this, we employ a pseudodifferential factorization of the wave operator that incorporates fractional attenuation. This factorization enables an approximate solution to the Helmholtz equation using a one-way (transmission) sweeping scheme, which is well-suited for high-frequency wave fields. We incorporate the three leading terms in the pseudodifferential expansion: the square-root first-order symbol representing wave propagation, the zeroth-order symbol accounting for amplitude modulation due to variations in wave speed and damping, and the next-order symbol capturing fractional attenuation. To improve computational efficiency and accuracy, we introduce wide-angle Padé approximations for the pseudodifferential operators corresponding to these three principal symbols. Finally, we present proof-of-concept numerical implementation of the proposed method and its application for medical imaging.

The inverse photoacoustic tomography problem in fractionally attenuating media

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Keywords: Inverse problems; photoacoustic tomography; attenuating media; fractional derivative

The inverse photoacoustic tomography (iPAT) problem involves determining the initial conditions of waves that propagate through a domain and are measured at its boundary. This problem arises in the medical imaging technique of the same name, where a biological tissue is illuminated by an electromagnetic pulse, generating ultrasound waves that are detected at the boundary of a region of interest and used to infer optical properties.

In biological tissues, it is well known that ultrasound waves experience attenuation, which typically follows a power-law dependence on frequency. This behavior is often modeled using fractional derivatives. In this talk, I will present recent results on uniqueness, stability, and reconstruction for the iPAT problem in attenuating media, where the attenuation is represented by a lower-order temporal Caputo derivative. This work extends and complements recent results from [1]. We will show that uniqueness is established through an approach first introduced in [2] and later adapted to the iPAT setting in [3], which combines two distinct types of Carleman estimates, while stability is obtained in a similar fashion as in [3], via a compact-perturbation argument that in this case requires establishing precise continuous dependence estimates involving Caputo fractional derivatives. Our analysis is based on the generalized formulation of the Caputo derivative developed in [4], together with a microlocal representation of solutions to the wave equation. Finally, assuming complete data and small attenuation, we achieve reconstruction via energy estimates and a Neumann series expansion, following a strategy similar to that in [3].

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Analysis of the anisotropic elastic tensor recovery problem from internal data

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Keywords: Inverse parameters problems; Elliptic PDEs; Elastography

Elastography is a medical imaging technique that exploits variations in tissue stiffness to detect early disease. From the measurement of several internal tissue displacements $\boldsymbol{u}$, the aim is to recover the elasticity tensor $\boldsymbol{C}$, which satisfies the linear elasticity equation $-\mathrm{div}(\boldsymbol{C} : \mathcal{E}(\boldsymbol{u})) = \boldsymbol{f}$. In the weak sense, this reads

$$\text{Find } \boldsymbol{C} \quad \text{s.t.} \quad \int_\Omega \boldsymbol{C} : \mathcal{E}(\boldsymbol{u}) : (\nabla \boldsymbol{v})^T = \langle \boldsymbol{f}, \boldsymbol{v} \rangle_{H^{-1}, H_0^1}, \quad \forall \boldsymbol{v} \in H_0^1(\Omega, \mathbb{R}^d). \tag{RWF}$$

Using this Reverse Weak Formulation, introduced in [1, 2], we analyze the stability of the inversion under minimal assumptions on the data.

Our approach is to transform the whole problem into a first-order linear system of the form $\nabla \boldsymbol{\mu} + \boldsymbol{B} \cdot \boldsymbol{\mu} = F$, where $\boldsymbol{B}$ and F are tensor fields constructed from the data. Using this form, we prove a closed range property and derive L^2 Lipschitz stability estimates. We then study the null space of this operator by introducing the notion of conservative tensor fields, constructed as a generalization of conservative vector fields.

We show that a suitable discretization of (RWF) allows a stable reconstruction of the parameter maps in dimension two. The simulations cover both isotropic and anisotropic cases under static and time-harmonic excitation.

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PINNs-based blood flow data assimilation from PC-MRI data with artifacts

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Keywords: Magnetic resonance imaging (MRI); blood flow; Stokes; Navier-Stokes; Physics-informed neural networks (PINNs)

Phase contrast magnetic resonance imaging (PC-MRI) is a technique used to measure blood velocity from a complex signal. It is a non-invasive technique for the diagnosis of cardiovascular diseases. However, there is an important limitation: the aliasing and noise in the image are traded off by the MRI scanner model for the encoded velocity. Such a model is defined by a parameter called Venc, which limits the maximum velocity that can be measured without suffering from aliasing while controlling how noisy the velocity is. In this context, aliasing consists of jump discontinuities in the acquired image due to the PC-MRI method for the velocity calculation, which wraps the resulting phase subtraction [1].

In this work, we study some strategies to deal with the aliasing problem with low-Venc PC-MRI measurements and show preliminary results of a new proposed method based on considering a fluid model. We present a functional that incorporates the aliased measurements and the model, inspired by the functional defined in [2] and the usual data assimilation functionals [3]. We analyze its convexity and uniqueness properties, which can provide a corrected velocity field and a recovered pressure. Finally, we show numerical experiments with PINNs [4] that motivate the applicability of the velocity correction method and also the pressure estimation.

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Data-driven multi-agent modelling of calcium interactions in cell culture: PINN vs Regularized Least-squares

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Keywords: data-driven methods, inverse problem; regularized least-squares model, physics-informed neural network.

Data-driven discovery of dynamics in biological systems allows to better observe and characterize processes, such as calcium signaling in cell culture. Recent advancements in techniques enable the exploration of previously unattainable insights of dynamical systems, overcoming the limitations of more classic methodologies, such as the Sparse Identification of Non-Linear Dynamics (SINDy). The latter require some prior knowledge of an effective library of candidate terms, which is not realistic for real case study. Using inspiration from fields like traffic density estimation and control theory, we propose a methodology for characterization and performance analysis of calcium delivery in a family of cells. In this work we compare performance of Constraint Regularized Least-Squares Method (CRLSM) and Physics-Informed Neural Networks (PINN) for system identification and parameter discovery for governing ordinary differential equations (ODEs). The CRLSM achieves fairly good parameter estimate and good data fit, when using the learned parameters in the Consensus problem. On the other hand despite the initial hypothesis, PINNs fail to match the CRLSM performance and under the current configuration do not provide fair parameter estimation. However, we have only studied limited number of PINN architectures and it is expected that additional hyperparameter tuning as well as uncertainty quantification could significantly improve the performance in future works.

Discovering Partially Known Ordinary Differential Equations: A Case Study On The Cellulose Degradation

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Keywords: System Identification; Inverse Problem, Physics-Informed Neural Networks; Symbolic Regression; Chemical Kinetics

The degree of polymerization (DP) is one of the methods for estimating the aging of polymer-based insulation systems, such as cellulose insulation in power components [1]. The main degradation mechanisms in polymers are hydrolysis, pyrolysis, and oxidation [2]. These mechanisms combined cause a reduction of the DP. However, data availability for these types of problems is usually scarce. Moreover, the equations and parameters describing the degradation mechanisms are based on empirical experiments. Therefore, modeling the degradation of cellulose insulation requires solving differential equations with partially known parameters. Their unknown components can consist of parameters, entire functions, or a combination of parameters and functions. One of the sets of Ordinary Differential Equations (ODEs) used to analyze the insulation aging with cellulose degradation data is Emsley's system of ODEs [3]. We recover the governing equations of the degradation system using a combination of Physics-Informed Neural Networks (PINNs) [4] and Symbolic Regression [5]. In our case, we assume that the differential equation modeling the DP values is unknown. Moreover, we also assume the value of the rate constant at which the initial reaction rate deteriorates is unknown. We employ PINNs with an extra network to approximate the values of the unknown function while estimating the unknown parameter. Finally, we rediscover the mathematical expression of the unknown function using Symbolic Regression [6]. The resulting model allows for discovering unknown parameters while accurately approximating the unknown function from the limited dataset. We discuss the challenges of exploring hyperparameters of the PINN model, fitting the shape of the unknown function to the suitable mathematical expression, and choosing operations and hyperparameters in the Symbolic Regression model. For the discovery of the function, the results generate different functional expressions which can lead to more precise identification of the equations that describe the degradation and aging mechanisms.

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Self-Supervised Deep Equilibrium Learning for Sparse-Angle CT Reconstruction

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Keywords: self-supervised learning; X-ray tomography; deep equilibrium

Deep learning has emerged as a powerful tool for solving inverse problems in imaging, including computed tomography (CT). However, most approaches require paired training data with ground truth images, which can be difficult to obtain, e.g., in medical applications. In this talk, I will present TomoSelfDEQ, a self-supervised Deep Equilibrium (DEQ) framework for sparse-angle CT reconstruction that trains directly on undersampled measurements. We establish theoretical guarantees showing that, under suitable assumptions, our self-supervised updates match those of fully-supervised training with a loss including the (possibly non-unitary) forward operator like the CT forward map. Numerical experiments on sparse-angle CT data confirm this finding, also demonstrating that TomoSelfDEQ outperforms existing self-supervised methods, achieving state-of-the-art results with as few as 16 projection angles.

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A uniqueness theorem for the Calderón problem with small anisotropies

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We study an anisotropic Calderóns problem of recovering potentials by the boundary measurements. In particular, we consider the Laplace-Beltrami operator for small metric perturbations of the Euclidean metric. Under a sign condition on the potential, we prove that the Dirichlet-to-Neumann map associated with the Schrödinger operator determines the potential. Furthermore, we establish a Lipschitz stability estimate for the inverse problem. The talk is based on a joint work with J.-N. Wang.

Momentum ray transform and its applications to polyharmonic inverse problems

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Keywords: Calderon problem, polyharmonic operators, momentum ray transform.

The momentum ray transform integrates a scalar function or a symmetric tensor field along straight lines, weighted by t^k for all non-negative integers k. This transform was introduced by Sharafutdinov [1, Chapter 2]. In this talk, we will explore key properties of this transform and its applications in solving certain Calderón type inverse problem for polyharmonic operators.

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An inverse Signorini obstacle problem

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Keywords: Inverse Obstacle Problem; Signorini boundary problem

We study the inverse problem of determining a Signorini obstacle from boundary measurements for the isotropic elasticity system. We prove that the obstacle can be uniquely determined by a single measurement of displacement and normal stress for the Signorini problem on an open subset of the boundary up to a natural obstruction. In addition to considering the Signorini problem, we develop techniques that can be used to study inverse problems for general differential inequalities. This is a joint work with Maarten V. de Hoop, Matti Lassas, Lauri Oksanen and Jinpeng Lu .

An inverse problem in anisotropic elasticity

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Keywords: elastic wave equation, anisotropic stiffness tensor

I will talk about the elastic wave equation that models waves that propagate through elastic media. We are particularly interested in materials that are highly anisotropic and aim at recovering the so-called stiffness tensor, that captures the elasticity of the material, from exterior measurements. For the inverse problem that we consider the exterior measurements correspond to the Dirichlet-to-Neumann map. The stiffness tensor is modelled as a fourth rank tensor. In the talk I will touch upon how the tensor behaves under coordinate transformations (that maintain the symmetries of the tensor), conformal freedoms between the parameters in the elastic wave equation and how these aspects affect whether the stiffness tensor can be recovered uniquely at the boundary.

A nonlinear inverse problem with applications in cardiac electrophysiology

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Keywords: Nonlinear boundary value problem; cardiac electrophysiology

The monodomain model of cardiac electrophysiology consists in a nonlinear reactiondiffusion equation coupled with an ordinary differential equation and regulates the behaviour of the transmembrane potential in the heart tissue. We investigate the inverse problem of identifying perfectly insulating regions within the cardiac tissue that represent ischemic areas. We prove that the geometry and location of these insulating regions can be uniquely determined using only partial boundary measurements of the transmembrane potential.

Passive inverse problems: stability and neural network solutions

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Keywords: Passive inverse problems; Stability; Machine learning

Linear inverse problems have been successfully handled since at least Tikhonovs seminal work. However, PDE based nonlinear inverse problems such as coefficient or inner geometry recovery are mathematically more involved. In such problems, the PDE usually includes a known forcing term and a feature to be recovered using overdetermined boundary data.
In applications, passive inverse problems may occur in radar imaging or seismology. In related mathematical models, the forcing term in the PDE is unknown while the geometry of a feature has to be recovered. In this sense, there are both linear and nonlinear unknowns in passive inverse problems. In this presentation, we will focus on examples of passive inverse problems illustrating the recovery of cracks in unbounded domains. We will first discuss recent results regarding the stability of the Hausdorff distance between cracks in terms of overdetermined boundary data [1]. Next, we will turn to the case where cracks are defined through a parameter m in $\mathbf{R}^p$ while the forcing term for the PDE is still in an infinite dimensional space [2]. In a recent study, we proved Lipschitz continuity of a related inverse operator if the forward operator is restricted to m -dependent finite dimensional spaces. These finite dimensional spaces are spanned by m -dependent singular functions which can be computed in practice. This led us to build neural networks that can numerically solve the crack inverse problem. The solution is computed in an efficient non-iterative way and is robust to noise.
This is joint work with S. C. Hawkins, M. Ganesh, and F. Triki.

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Extension problem for the fractional parabolic Lamé operator and unique continuation

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Keywords: Fractional parabolic Lamé operator, Extension problem, Unique Continuation

In this talk, we will discuss an explicit formulation of fractional powers of the parabolic Lamé operator $\mathbb{H}$. We will then study the extension problem associated to such non-local operators which are closely related to the fractional heat operator. For $s \geq 1/2$, a space-like strong unique continuation result will be presented for the operator $\mathbb{H}^s + V$ by proving a conditional doubling property for solutions to the corresponding reduced system followed by a blowup argument. This has potential applications for inverse problems related to fractional operators in time-domain.

This talk will be based on the work [1].

Bibliography

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The Calderón problem for nonlocal parabolic operators

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Keywords: Calderón problem; Nonlocal parabolic operators

We investigate inverse problems in the determination of leading coefficients for nonlocal parabolic operators, by knowing the corresponding Cauchy data in the exterior space-time domain. The key contribution is that we reduce nonlocal parabolic inverse problems to the corresponding local inverse problems with the lateral boundary Cauchy data. In addition, we derive a new equation and offer a novel proof of the unique continuation property for this new equation. We also build both uniqueness and non-uniqueness results for both nonlocal isotropic and anisotropic parabolic Calderón problems, respectively.

Stability of time-independent coefficients of relativistic Schrödinger equation in infinite waveguide

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Keywords: Relativistic Schrödinger equation; infinite wave guide

In this talk, we address the stability of the inverse problem concerning the determination of time-independent vector and scalar potentials in a relativistic Schrödinger equation in an infinite waveguide using the measurement of Dirichlet to Neumann map. We will demonstrate that Hölder-type stability estimates can be established for both the scalar and vector potentials, up to a natural gauge invariance.

Sample error estimates for sparsity-promoting learned regularizers

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Keywords: Learned regularization; Sample error estimates; Sparsity promotion

Data-driven methods for inverse problems are a key tool to leverage large datasets and statistical learning techniques, resulting in reconstruction algorithms that successfully encode prior information about the solution. The theoretical investigation of such techniques has attracted growing interest, e.g., on the topics of stability and explainability. In this talk, I will address the generalization properties of data-driven reconstruction methods, focusing on the analysis of the sample error. This task, well-developed in statistical learning, is to estimate the dependence of the learned operators with respect to the data employed for their training.

To do so, I will first describe a rather general strategy, developed in [1], which is based on statistical learning and functional analysis techniques, and whose assumptions are met for a large class of inverse problems and supervised learning methods. Then, I will focus on the task of learning a synthesis operator in sparsity-promoting regularization, which is the object of [2]. After showing that this challenging problem falls within the proposed framework, I will carefully verify the related assumptions and discuss some relevant applications.

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Invertible Neural Operators and their Deep Theory

Michael Puthawala[1], Takashi Furuya[2], Matti Lassas[3], and Maarten de Hoop[4]

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Keywords: Machine Learning, Neural Operators, Functional Analysis

Neural Operators are an important and practical method for learning operators between Hilbert spaces. They are especially well-suited for scientific computing and numerical analysis, where they are used to directly solve PDE, shortcut expensive numerical solvers, solve inverse problems, and more. In this walk, we will do a deep dive on recent work on the theory at the foundation of neural operators. We will see how the desire to use injective and invertible neural operators for inverse problems leads to novel and deep questions in functional analysis. Finally, we will see how the desire to approximate invertible Hilbert operators with neural operators leads to a no-go theorem because of a surprising Hilbert-hotel type paradox. We will also see how to step around this fundamental obstruction in cases when the Hilbert operator is monotone or bilipschitz.

On the anisotropic fractional Calderón problem with exterior data

Ali Feizmohammadi[1], Tuhin Ghosh[2], Katya Krupchyk[3], Angkana Rüland[4], Johannes Sjöstrand[5], and Gunther Uhlmann[6]

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Keywords: fractional Calderón problem, uniqueness, anisotropic

In this talk, I discuss uniqueness up to natural gauges for the fractional anisotropic Calderón problem. A key difficulty consists in reconstructing the heat kernel, which relies on regularity estimates for the nonlocal problem. This talk is based on joint work with A. Feizmohammadi, T. Ghosh, K. Krupchyk, J. Sjöstrand and G. Uhlmann

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Calderón problem for fractional Schrödinger operators on closed Riemannian manifolds

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In this talk, we will discuss an analog of the anisotropic Calderón problem for fractional Schrödinger operators on closed Riemannian manifolds of dimension two and higher. We will demonstrate that the knowledge of a Cauchy data set of solutions to the fractional Schrödinger equation, given on an open nonempty subset of the manifold, determines both the Riemannian manifold up to an isometry and the potential up to the corresponding gauge transformation, under certain geometric assumptions on the manifold as well as the observation set. This is joint work with Ali Feizmohammadi and Gunther Uhlmann.

A linearized montonicity method for elastic waves

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Keywords: Monotonicity Method; Elasticity

In this talk I will present a linearized version of the monotonicity method for shape reconstruction using time harmonic elastic waves. An important advantage of a linearized method is that it provides an efficient version of the method that drastically reduces computation time. Our method has interestingly the additional advantage that it can be used to obtain further information on the material parameters, and is able to partially separate and identify the supports of the Lamé parameters.

Stability of inverse problems arising from wave equations of Magnetic Schrödinger operators

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Keywords: Magnetic Schrödinger; Stability estimates; Hyperbolic inverse problems

I will present on-going work regarding Hölder stability estimates for two inverse problems arising from the wave equation associated to a Magnetic Schrödinger operator on a simple Riemannian manifold. The first such inverse problem is the question of stably recovering a non-negative electric potential, and the solenoidal part of a magnetic potential from measured Neumann boundary observations; the second problem similarly studies the question of stably recovering such potentials from the boundary spectral data of the associated Magnetic Schrödinger operator. I will discuss the connection between these two problems, and the technical issues introduced by the magnetic potential. This work is joint with Boya Liu, Teemu Saksala, and Lili Yan.

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