

4. $F(x, y, z) = (0, y, z)$.

6. a) $\text{Ker}(F) = \{(0, 0, 0)\}$. Logo F é bijetor (invertível). $F^{-1}(x, y, z) = (x + 3y + 14z, y + 4z, z)$.

7. $F(x, y, z) = \left(x + y - \frac{1}{2}z, x + y + \frac{3}{2}z\right)$ é inversível e $F^{-1}(x, y, z) = \left(y, \frac{1}{4}(3x - 4y + z), \frac{1}{2}(x - z)\right)$.

10. $F(x_1, \dots, x_n) = (x_1, x_2, \dots, x_{n-1})$ é inversível e $F^{-1}(x_1, \dots, x_n) = (x_1, x_2, \dots, x_n)$.

5.1. 1. $(F + H)(x, y) = (x, x + 2y)$;

$(F \circ G)(x, y) = (y, 2x + 2y)$;

$(G \circ (F + H))(x, y) = (x + 2y, 2x + 2y)$.

2. $(F \circ G)(x, y, z) = (x + 3y - z, x + y + z, x + 2z)$;

$\text{Ker}(F \circ G) = \{(y, -2, 1, 1) \mid y \in \mathbb{R}\}$;

$\text{Im}(G \circ F) = \{(x, 0, 1) + y(3, 1, 1) \mid x, y \in \mathbb{R}\}$.

4. Se n é ímpar, $F^n(x, y) = (y, x)$ e $F^n(x, y) = (x, y)$ se n é par.

5. $(1 + F + F^2)(x, y) = (22x + 21y, 35x + 36y)$ não é isomorfismo pois $\text{Ker}(1 + F + F^2) = \{(x, -x) \mid x \in \mathbb{R}\}$.

11. 2) $(1 - F)(x, y, z, t) = (x, y - x, z - y - 2x, t - z - 2y - 3x)$ é isomorfismo pois $\text{Ker}(1 - F) = \{(0, 0, 0, 0)\}$.

12. 1) $F^2(z) = i$;

3) $G^2(z) = -z$;

13. 1) e 2) Nenhuma das duas coisas;

3) Idempotente;

4) Nilpotente.

14. 1) $F^2(x, y) = (x, 2x + y)$;

2) $(F - I)(x, y) = (0, x)$.

19. Para todo $f(t) \in P_n(\mathbb{R})$, $D^{n+1}(f(t)) = 0$.

5.5. 1. $(F) = \begin{pmatrix} 2 & 1 & 9 \\ 11 & 11 & 11 \\ 10 & 6 & 10 \\ 11 & 11 & 11 \end{pmatrix}$

2. 1) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; 2) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$;

9. $\langle A, B \rangle = 1$, $\|A\| = \sqrt{3}$, $\|B\| = 1$ e $d(A, B) = \sqrt{2}$.

3) $\begin{pmatrix} 2 & 1 & -1 & 3 \end{pmatrix}$; 4) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

4. $(F)_{\mathbb{B}} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix}$.

Logo o traço de $(F)_{\mathbb{B}}$ é 4.

6. $\begin{pmatrix} 6 & -1 \\ +20 & -4 \end{pmatrix}$.

8. $F(x, y) = \frac{1}{5}(13x + y, 86x - 3y)$.

10. (I) $\begin{pmatrix} 2 & 0 & \frac{2}{3} \end{pmatrix}$;

(II) $\begin{pmatrix} -1 & -1 & \frac{2}{3} \end{pmatrix}$.

12. $F(x, y) = (x, y)$, $F(x, y) = (x, bx)$,

$F(x, y) = (0, bx + y)$ e $F(x, y) = (0, 0)$.

14. $(G) = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 0 \\ -1 & 0 & -3 \end{pmatrix}$ e $G(x, y, z) = (x - y, 3y, -x - 3z)$.

5. Apend. 1. $(F_1 + F_2)(x, y, z) = 3x - 4y + 7z$;

$(2F_1 + 3F_2)(x, y, z) = 8x - 9y + 7z$.

3. (I) $\{v_1, v_2, v_3\}$, onde $v_1(x, y, z) = \frac{1}{2}z$, $v_2(x, y, z) = -2x + \frac{3}{2}y + \frac{1}{4}z$ e $v_3(x, y, z) = x - \frac{1}{2}y - \frac{1}{4}z$;

(IV) $\{v_1, v_2, v_3\}$, onde $v_1(a + bt + ct^2) = a + c$, $v_2(a + bt + ct^2) = b$ e $v_3(a + bt + ct^2) = -c$.

4. $\{(2, 2, 1)\}$.

7. (I) Sim; (II) Sim.

10. Sim.

12. $a = b = c = 0$.

6.3. 1. 1) $k > 9$.

3. $\langle f(t), g(t) \rangle = -\frac{5}{3}$, $\|f(t)\| = \sqrt{\frac{331}{210}}$, $\|g(t)\| = \sqrt{\frac{28}{15}}$ e $\|f(t) + g(t)\| = \sqrt{\frac{23}{210}}$.

4. Sim.

7. $\alpha > 0$.

10. $\langle u, v \rangle = 7$, $\|u\| = \sqrt{6}$, $\|v\| = \sqrt{30}$, $d(u, v) = \sqrt{\frac{u+v}{\|u\| + \|v\|}} = \frac{1}{5\sqrt{2}}(4, 3, 4, 3)$

$\cos(u, v) = \frac{7}{6\sqrt{5}}$

12. $\frac{1}{2}$.

14. 3) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 4) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

15. $\alpha = \pm \frac{3}{5}$.

18. (I) $\|u\| = \sqrt{15}$; (II) $\|f(t)\| = \sqrt{\frac{31}{30}}$;

(III) $\|A\| = \sqrt{10}$.

20. (I) $d(u, v) = \sqrt{2}$, $\cos(u, v) = \frac{\sqrt{2}}{2}$.

(III) $d(A, B) = \sqrt{2}$ e $\cos(A, B) = \frac{1}{2}$.

22. Se $e_1 \neq e_j$, $\cos(e_1, e_j) = \frac{1}{2}$ e $\cos(e_1, e_j) = 1$ se $e_1 = e_j$.

6.6. 1. 1) $m = 1$ ou $m = 6$;

2) $(0, x)$, $\forall x \in \mathbb{R}$;

3) $(1, 0)$.

3. 1) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right)$; $\left(-\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}\right)$, 0 ; $\left(\frac{2}{\sqrt{153}}, \frac{-2}{\sqrt{153}}, \frac{1}{\sqrt{153}}, \frac{12}{\sqrt{153}}\right)$.

4. $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right); (0, 0, 1)\right\}$.

5. $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$.

7. 1) $\left\{1, 2\sqrt{3}t - \sqrt{3}, \sqrt{5}(6t^2 - 6t + 1)\right\}$.

2) $\left\{-\frac{b}{6}t^2 + bt - b \mid b \in \mathbb{R}\right\}$.

8. De W : $\left(\frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, 0\right)$;

$(0, 0, 0, 1)$.

De $W^\perp$: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right)$; $\left(\frac{3}{\sqrt{22}}, \frac{-3}{\sqrt{22}}, \frac{2}{\sqrt{22}}, 0\right)$.

$\left(\frac{6}{7}, \frac{9}{7}, \frac{-2}{7}, \frac{-1}{7}\right)$.

10. $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$; $\left(\frac{1}{\sqrt{18}}, \frac{-2}{\sqrt{18}}, \frac{3}{\sqrt{18}}\right)$.

13. $g(t) = \frac{1}{20}t + \frac{3}{2}t^2 - t^3$.

19. 1) $v_1 = \frac{1}{3}(2, 2, -4)$ e $v_2 = \frac{1}{3}(7, 7, 7)$,
2) $v = \frac{1}{7}(18, 15, 16) + \frac{1}{7}(3, 6, -9)$.

22. $m = \frac{1}{\sqrt{3}}$.

23. $\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$.

25. $\frac{1}{13} \begin{pmatrix} -12 & -5 \\ 5 & -12 \end{pmatrix} = B$

27. $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$

7.2. 1. a) 1;

b) 1;

c) 1;

d) -1.

2. a) 1;

b) -2;

c) -3;

d) 2;

e) -8;

f) 24.

4. a) $-\lambda(\lambda + 2)$; b) $(2 - \lambda)(\lambda^2 - 3\lambda - 1)$.

5. a) $\lambda = 0$ ou $\lambda = -2$;

b) $\lambda = 2$ ou $\lambda = \frac{3 \pm \sqrt{13}}{2}$.

6. $p(\lambda) = (2 - \lambda)(\lambda^2 - 3\lambda - 1)$ e $p(A) = 0$.

7.3. 4. São iguais.

6. Zero.

7.4. 1. $A_{11} = -5$, $A_{12} = 4$, $A_{13} = -2$; $A_{21} = 1$, $A_{22} = -1$, $A_{23} = 1$, $A_{31} = 3$, $A_{32} = -2$ e $A_{33} = 1$.

2. -3.

3. $3 \cdot \det \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} - \det \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} + 2 \cdot \det \begin{pmatrix} 4 & 0 \\ 1 & 0 \end{pmatrix} = 2$.

4. $2 \cdot \det \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix} - \det \begin{pmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix} +$