# Brazilian Conference on Operator Algebras

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#### Johannes Aastrup

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**Title:** A continuous Field of  $C^*$ -algebras and the Tangent Groupoid for Manifolds.

**Abstract:** Joint work with R. Nest and E. Schrohe. For a smooth manifold with boundary we construct a semigroup and a continuous field asociated with this semigroupoid which extend Connes' construction of the tangent groupoid.

#### Massoud Amini

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Department of Mathematics, Tarbiat Modares University, Tehran, Iran.

Title: Amenablity of universal groupoids.

**Abstract:** Necessary and sufficient condition on an inverse semigroup is given so that its universal groupoid is amenable.

#### Martin Argerami

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**Title:** A Schur-Horn Theorem in  $II_1$  factors.

**Abstract:** The classical Schur-Horn Theorem characterizes the relation between the diagonal and the eigenvalues of a selfadjoint matrix. In a recent preprint, Arveson and Kadison left as an open problem a Schur-Horn Theorem in  $II_1$  factors, where the role of the diagonal matrices is played by a masa. We will discuss a version of the Theorem obtained jointly with Pedro Massey.

# **Farkhad Nematjonovich Arzikulov** arzikulovfn@yahoo.com Andizhan State University, Andizhan city, Uzbekistan.

**Title:** Type and Peirce decompositions for C\*-algebras.

**Abstract:** It is known that any two elements a, b of a C<sup>\*</sup>-algebra A, who satisfy the conditions  $a \ge 0$ , ab + ba = 0 are pairwise commutative, i.e. ab = ba. This fact allows us to use *Jordan annihilators*, defined as  $Ann(S) = \{a \in A : as + sa = 0, s \in S\}$ , instead of projections in the case of general C<sup>\*</sup>-algebras to research some structure and classification problems of the theory of C<sup>\*</sup>-algebras. For this propose we used the set  $\mathcal{P}$  of all Jordan annihilators of subsets of positive elements of a C<sup>\*</sup>algebra. This set forms a complete lattice. We developed some theory of lattices of Jordan annihilators. In particular we introduced and studied such notions as orthogonal annihilators, central annihilators and equivalent annihilators. For example, an annihilator  $V \in \mathcal{P}$  is called central if  $^{d}(Ann(Ann(S))) \cap ^{d}(Ann(S)) = 0$ , where  $S \subseteq A_{+}, V = Ann(Ann(S))$  and  ${}^{d}V = \{a \in A : xay + yax = 0 \text{ for all } x, y \in V\}$ . The set of all central annihilators of the lattice  $\mathcal{P}$  forms a complete sublattice. Then we introduced notions of C<sup>\*</sup>-algebras of types I,  $II_1$ ,  $II_{\infty}$  and III. Using these notions we have established that any C<sup>\*</sup>-algebra A has unique C<sup>\*</sup>-subalgebra B, who is a direct sum of  $C^*$ -subalgebras of types I, II and III, which are maximal in A annihilators of types I, II and III accordingly. For the decomposition formula to be established we introduced also the notion of algebraically density of a subset in the C<sup>\*</sup>-algebra (a subset B of a C<sup>\*</sup>-algebra A is algebraically dense in A, if Ann(B) = 0). The next statement takes place: Let  $S \subset A_+$ . We say that S separates a subset F of the space  $A^*$ , if for any  $\rho \in F$  there exists  $s \in S$  such that  $\rho(s) \neq 0$ . Then for any C<sup>\*</sup>-algebra A and subset  $S \subset A_+$ the set  $Ann(S) \cup S$  separates the space  $A^*$  if and only if the set  $Ann(S) \cup S$ is algebraically dense in w(A). It is showed that not for any C<sup>\*</sup>-algebra A and subset  $S \subset A_+$  the set  $Ann(S) \cup S$  is algebraically dense in A \* \* (or in w(A)). Then the classification decomposition theorem can be formulated as follows: Let A be an C<sup>\*</sup>-algebra of linear operators on Hilbert space H, w(B) be a weak-\* closure of a subset  $B \subseteq B(H)$ . Then there exist such unique C<sup>\*</sup>-subalgebras  $A_I$ ,  $A_{II}$ ,  $A_{III}$  of A of types I, II and III respectively that  $A_I \oplus A_{II} \oplus A_{III}$  is a C\*-subalgebra of A, and  $w(A) = w(A_I) \oplus w(A_{II}) \oplus w(A_{III})$ if and only if  $A_I \oplus A_{II} \oplus A_{III}$  is algebraically dense in w(A). Then we gave an example when  $A_I \oplus A_{II} \oplus A_{III} \neq A$ . Then we studied the connection

between these notions of types and the known notions of "finite", "infinite", "properly infinite" and "purely infinite". For example, a simple C\*-algebra is purely infinite if and only if this C\*-algebra is of type III and any purely infinite C<sup>\*</sup>-algebra is of type III. In the last In the article "On Jordan algebras of Bear type" (Bladikavkaz Mathematical Journal, www.vmj.ru, No 3, V. 4, 2002) we have realized an idea of construction of an infinite Peirce decomposition in case of monotone complete JB-algebras. It is clear that direct summands of any Peirce decomposition are Jordan annihilators. In the work we introduced a notion of infinite Peirce decomposition in case of general C<sup>\*</sup>algebras. We established that if the infinite Peirce decomposition  $\sum_{\xi\eta}^{\oplus} p_{\xi} A p_{\eta}$ of a C<sup>\*</sup>-algebra A with an infinite orthogonal set  $\{p_{\xi}\}$  of projections with the least upper bound 1 in A is monotone complete then this infinite Peirce decomposition is a C<sup>\*</sup>-algebra about naturally introduced associative multiplication and the norm. Conversely if projections of the set  $\{p_{\xi}\}$  are pairwise equivalent and  $\sum_{\xi\eta}^{\oplus} p_{\xi} A p_{\eta}$  is a C\*-algebra then  $\sum_{\xi\eta}^{\oplus} p_{\xi} A p_{\eta}$  is monotone complete. In here we used the constriction of infinite Peirce decomposition for monotone complete algebras. Then we introduced a notion of infinite Peirce decomposition using a constriction of infinite Peirce decomposition for simple algebras. We proved that any C<sup>\*</sup>-factor of type I can be represented as an infinite Peirce decomposition for monotone complete C<sup>\*</sup>-algebras or as an infinite Peirce decomposition  $\sum_{\xi\eta}^{o} p_{\xi} A p_{\eta}$  for simple C\*-algebras. If a C\*-factor A has an infinite Peirce decomposition  $\sum_{\xi\eta}^{o} p_{\xi} A p_{\eta}$  then this algebra A is simple. At the end of the work some prospects of the work are fixed.

#### Alexandre Baraviera

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**Title:** Invariant probabilities for subshifts on countable alphabets.

**Abstract:** Motivated by the study of traces of certain operator algebras, we investigate the existence of finite invariant measures (probabilities) for sub-shifts of finite type on countable alphabets. The measures we are looking for are the eigenmeasures of the dual of a transfer operator associated to the shift and depends on a given potential function. We state some assumptions on this potential that assure the existence of the desired measures.

#### Valentin Deaconu

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Title: C\*-algebras associated with interval maps.

Abstract: For each piecewise monotonic map  $\tau$  of [0, 1], we associate a pair of C\*-algebras  $\mathcal{F}_{\tau}$  and  $\mathcal{O}_{\tau}$ , and calculate their K-theory, getting invariants for  $\tau$ . When these algebras are simple,  $\mathcal{F}_{\tau}$  has a unique trace, and  $\mathcal{O}_{\tau}$  is purely infinite with a unique KMS-state. In the case that  $\tau$  is Markov, our algebras include the Cuntz-Krieger algebras  $\mathcal{O}_A$ , and the associated AF-algebras  $\mathcal{F}_A$ . Other examples are related to tent maps, quadratic maps, multimodal maps, interval exchange maps, and  $\beta$ -transformations. For the last two kinds of maps, the C\*-algebra  $\mathcal{O}_{\tau}$  coincides with the algebras defined by Putnam and Katayama-Matsumoto-Watatani respectively.

#### Carla Emilia Farsi

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**Title:** Orbifold Index Cobordism Invariance. **Abstract:** We prove a K-theoretical Index Cobordism Invariance Theorem for G-transversally elliptic operators. (Here G is an even-dimensional torus.) This result implies index cobordism invariance for orbifolds.

#### Miguel Ferrero

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**Title:** Ações parciais de tipo finito.

Abstract: Definiremos ações parciais de tipo finito de grupos em álgebras e apresentaremos propriedades destas ações. Estudamos tambem ações parciais sobre álgebras finitamente geradas e projetivas.

Pinhas Grossman pinhas@math.berkeley.edu Department of Mathematics University of California at Berkeley Berkeley, CA, USA.

**Title:** Forked Temperley-Lieb Algebras and Intermediate Subfactors. **Abstract:** We consider noncommuting pairs P,Q of intermediate subfactors of an irreducible, finite-index inclusion N in M of II<sub>1</sub> factors such that P and Q are supertransitive with Jones index less than 4 over N. We show that up to isomorphism of the standard invariant, there is a unique such pair corresponding to each even value  $[P:N] = 4\cos^2(\pi/2n)$  but none for the odd values  $[P:N] = 4\cos^2(\pi/(2n+1))$ . The proof uses both the planar algebra apparatus for intermediate subfactors and an algebraic construction of a "forked" version of the Temperley-Lieb algebras, which captures the structure of noncommuting supertransitive subfactors with small index.

## Alexander A. Katz

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**Title:** Real and Jordan Structures in Locally C\*-algebras. **Abstract:** We disscuss real and Jordan structures in projective limits of complex C\*-algebras, and study their properties and representations.

#### Vladimir Manuilov

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Title: On the asymptotic tensor product of C\*-algebras.

Abstract: We define a tensor  $C^*$ -norm, which is the minimal  $C^*$ -norm that is functorial with respect to asymptotic homomorphisms. Some properties of this norm will be discussed, in particular, we show that this norm is not associative. Applications to the C\*-extensions theory will be given.

# Márcio Martins

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**Title:** The eigenspectrum of integrable ice-rule multistate vertex models. **Abstract:** The physical understanding of two-dimensional classical systems of statistical mechanics includes necessarily the exact diagonalization of their transfer matrices. Here we consider this problem for solvable vertex models whose edge states a,b,c,d takes value on the spin S set [-S, ..., S] such that the respective Boltzmann weights  $R_{ab}^{cd}$  are constrained by by the ice-rule a + b = c + d. We are able to formulate the quantum inverse scattering method for these lattice models form an unified point of view. This leads us to exhibit explicit and general formulas for the eigenvectors and the eigenvalues of transfer matrix depending only on the structure constants  $R_{ab}^{cd}$ .

#### Severino Toscano Melo

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Title: K-Theoretic proof of Boutet de Monvel's index theorem.

Abstract: Let A denote the norm closure of all Boutet de Monvel operators of order and class zero on a compact manifold with boundary X. Boutet de Monvel's index theorem for boundary value problems states that the Fredholm index for A, regarded as a map from K1(A/K) to Z (K denoting the compact ideal) factors through K0 of the cotangent bundle T\*X. The mapping from K(T\*X) to Z is Atiyah-Singer's topological index. The mapping from K1(A/K) to K(T\*X) was constructed by Boutet de Monvel (Acta, 1971) through a series of explicit deformations of elliptic boundary value problems. In a joint work with Thomas Schick and Elmar Schrohe (to appear in Crelle's journal), we show that a great deal of those explicit deformations can be substituted by abstract K-theory arguments. That also gives a better picture of where the mapping from K1(A/K) to K(T\*X) comes from. All hard analysis results needed for this proof are convenient descriptions of the kernel and the image of the boundary value symbol of Boutet de Monvel's calculus, obtained in a joint work with Ryszard Nest and Elmar Schrohe (Crelle's, 2003). The K-theory techniques employed in the more recent work also lead to an improvement of results about the K-theory of A we proved in the 2003 paper.

#### Marcela Irene Merklen

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**Title:** Abstract characterizations of Pseudodifferential Operators. **Abstract:** Cordes has proved that the class of all bounded adjointable operators on L2(Rn) which are smooth under the canonical action of the Heisenberg group coincides with the class of all pseudodiferential operators with symbol possessing bounded derivatives of all orders At the end of Chapter 4 of "Deformation Quantization for Actions of Rd", Rieffel has conjectured a similar characterization of a certain class of C\*-algebra-valued-symbol pseudos. We have proved (M. I. Merklen, "Boundedness of pseudodifferential operators of a C\*-algebra valued symbol") that, if the natural generalization of Cordes' characterization holds for pseudos with symbols taking values on a C\*algebra A, then Rieffel's conjecture is true for A. As an application (S. T. Melo, M. I. Merklen "C\*-algebra-valued-symbol pseudodifferential operators: abstract characterizations", in preparation), we have proven that Rieffel's conjecture is true for any separable commutative C\*algebra.

#### **Bertrand Monthubert**

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Title: A topological index theorem on manifolds with corners.

Abstract: We define an analytic index and prove a topological index theorem for a non-compact manifold  $M_0$  with poly-cylindrical ends. We prove that an elliptic operator P on  $M_0$  has an invertible perturbation P + R by a lower order operator iff its analytic index vanishes. As an application, we determine the K-theory groups of groupoid  $C^*$ -algebras of manifolds with corners. Alternative talk: Boutet de Monvel calculus and Groupoids This paper is part of our effort to answer the question: Can Boutet de Monvel's algebra on a compact manifold with boundary X be obtained as the algebra of pseudodifferential operators  $\Psi^0(G)$  on some Lie groupoid G? If it could, the kernel of the principal symbol would have to be isomorphic to the groupoid  $C^*$ -algebra  $C^*(G)$ . We exhibit a groupoid G such that  $C^*(G)$  possesses an ideal  $\mathfrak{I}$  isomorphic to  $\mathfrak{G}$ , the kernel of the principal symbol homomorphism on Boutet de Monvel's algebra. That follows from the fact that both  $\mathfrak{I}$  and  $\mathfrak{G}$  are extensions of  $C(S^*Y) \otimes \mathfrak{K}$  by  $\mathfrak{K}$  (where  $\mathfrak{K}$  denotes the compact ideal and  $S^*Y$  denotes the co-sphere bundle over the boundary of X), and that the index mappings in the standard K-theory cyclic exact sequences induced by these two extensions are the same.

#### Jens Mund

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Title: Modular localization and string-localized quantum fields.

Abstract: The concept of modular localization introduced by Brunetti, Guido and Longo, and Schroer, can be used to construct quantum fields. It combines Wigner's particle concept with the Tomita-Takesaki modular theory of operator algebras. I shall report on the construction of quantum fields which are localized in semi-infinite strings extending to spacelike infinity. (Joint work with B. Schroer and J. Yngvason.) Particular applications are: Wigner's massless "infinite spin" particles; Anyons in d=2+1 (massive particles with non-halfinteger spin); String-localized vector/tensor potentials for photons resp. gravitons. Some ideas will be presented concerning the perturbative construction of gauge theories completely within a Hilbert space, trading gauge dependence with dependence on the direction of the localization string.

#### Andrey Muravnik

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**Title:** On asymptotic closeness of solutions for differential and differentialdifference parabolic operators. **Abstract:** The Cauchy problem with a bounded continuous initial-value function  $u_0$  is considered for the equation

$$\frac{\partial u}{\partial t} = \Delta u + \sum_{i=1}^{n} \sum_{j=1}^{m_i} a_{ij} u(x_1, \dots, x_{i-1}, x_i + b_{ij}, x_{i+1}, \dots, x_n, t),$$
(1)

where  $m_i$  are natural, while  $a_{ij}$  and  $b_{ij}$  are real. Equations of the above kind arise in various models of mathematical physics (see, e.g., [1, 2, 3]). The classical unique solvability of the above problem is proved in [4], while the classes of uniqueness are found in [5]. We take the *differential-difference* operator standing at the right-hand part of Eq. (1) and replace all its nonlocal terms by their Taylor expansions up to the second-order term (i.e., the order of the equation). The obtained *differential* operator with constant coefficients is denoted by L. The main result presented is as follows: *if the operator*  $\frac{\partial}{\partial t} - L$  is parabolic, then the solutions of the above Cauchy problem for Eq. (1) and  $\left(\frac{\partial}{\partial t} - L\right)u = 0$  are asymptotically closed with the weight  $e^{-t\sum_{i=1}^{n}\sum_{j=1}^{m_i} a_{ij}}$ (note that this weight guarantees the boundedness of the latter solution).

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#### Justin R. Peters

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Department of Mathematics Iowa State University Ames, Iowa, U.S.A.

**Title:** Extensions of dynamical systems and C\*-envelopes of semicrossed products.

Abstract: Let X be a compact metric space, and  $\varphi : X \to X$  a continuous surjection. By an extension of the dynamical system  $(X, \varphi)$  we mean a compact metric space  $\tilde{X}$ , a continuous surjection  $\tilde{\varphi} : \tilde{X} \to \tilde{X}$  and a continuous surjection  $p : \tilde{X} \to X$  such that the diagram.... is commutative. It is possible to find an extension for which the map  $\varphi$  is a homeomorphism. The notion of C<sup>\*</sup>-envelope of an operator algebra is due to Arveson and Hamana. We consider the C<sup>\*</sup>-envelope of a semicrossed product of a commutative C<sup>\*</sup>-algebra with respect to an endomorphism. This has been done by Muhly and Solel and others, but here we take the point of view of dynamical systems. Specifically, let  $\alpha$  be an endomorphism of C(X) of the form,  $\alpha(f) = f \circ \varphi$ , where  $\varphi$  is a continuous surjection of the compact metric space X. Is the C<sup>\*</sup>-envelope of the associated semicrossed product identified with the crossed product of  $C(\tilde{X})$  with respect to  $\tilde{\varphi}$ , where  $(\tilde{X}, \tilde{\varphi})$  is the minimal extension of  $(X, \varphi)$  with  $\tilde{\varphi}$  a homeomorphism?

#### Kar-Henning Rehren

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**Title:** Classification results in conformal quantum field theory. **Abstract:** Using methods from Operator Algebras, the A-D-E classification of modular invariant matrices is used to classify chiral, two-dimensional, and boundary CFTs with c < 1.

# Jean Renault Jean.Renault@univ-orleans.fr Faculté des Sciences Université d'Orléans, Orléans, France.

Title: Regular masas in C\*-algebras.

Abstract: According to a well-known theorem of Feldman and Moore, the group measure space construction gives an equivalence of categories between twisted countable standard measured equivalence relations and Cartan pairs, i.e. a von Neumann algebra (on a separable Hilbert space) together with a Cartan subalgebra. Kumjian gave a C\*-algebraic analogue of this theorem in the early eighties. I will present an extension of Kumjian's theorem which includes Cuntz algebras and foliation algebras.

# Pedro Lauridsen Ribeiro

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**Title:** Return to Equilibrium, Horizons and Thermalization of Nonautonomous C<sup>\*</sup>- and W<sup>\*</sup>-dynamical Systems

**Abstract:** The most general deduction of the Hawking-Unruh temperature in quantum field theory in curved spacetime is done in the context of von Neumann algebras, by means of the KMS condition in its algebraic incarnation - namely, the proper time accelerated dynamics is dictated by the Tomita-Takesaki modular group, and the correct value of the temperature (determined by the KMS periodicity) is computed by means of a conformal (or, more generally, real-analytic) time reparametrization. Here we set out to study the more general problem of non-autonomous C\*- and W\*dynamics with possess a property of return to equilibrium; and such that the aforementioned time reparametrizations are no longer analytic. We shall show certain situations where the long-term thermalization occurs in the distant past/future, possibly at different temperatures. This construction can be applied to the study of the quantum thermodynamics of asymptotically stationary black holes, where the asymptotic temperature is fixed by the constrained reparametrization freedom at the horizon under certain conditions on the test quantum system, giving thus a microscopic derivation of the zeroth and first laws of black hole thermodynamics from a non-equilibrium perspective.

#### Mikael Rordam

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Title: Purely infinite C\*-algebras of real rank zero.

Abstract: In this talk, we present a recent result by C. Pasnicu and the author that a separable purely infinite C\*-algebra is of real rank zero if and only if its primitive ideal space has a basis of compact-open sets and the natural map  $K_0(I) \to K_0(I/J)$  is onto for all closed two-sided ideals J contained in I in the C\*-algebra. In particular, if A is any separable C\*-algebra, then A tensor  $O_2$  is of real rank zero if and only if the primitive ideal space of A has a basis of compact-open sets.

# Gil Schieber

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CBPF - Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, RJ, Brazil.

Title: Modular splitting method: from modular invariants to graphs.

#### Bert Schroer

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**Title:** Holography onto Nullsurfaces using modular operator algebra concepts.

**Abstract:** Whereas in the absence of interactions the holography of bulk quantum matter onto its null-horizon can be done in terms of elementary calculations, the presence of interactions requires the use of modular theory. There are interesting consequences for thermal behavior and symmetry properties.

## Martin Edward Walter

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#### Title: Duality for Groups.

**Abstract:** We report on the current status of our duality theory of groups – a theory based on order structures inspired by the theory of operator algebras.

#### Yasuo Watatani

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Title: C\*-algebras associated with complex dynamical systems.

Abstract: Iteration of a rational function R gives a complex dynamical system on the Riemann sphere. We introduce three C\*-algebras associated with R as Cuntz-Pimsner algebras of Hilbert bimodules over the algebras of continuous functions on the Riemann sphere, Julia set or Fatou set. We review relations between complex dynamical systems and the associated  $C^*$ -algebras. The algebra corresponding to the Julia set is purely infinite, simple C\*-algebra. We can recover the degree of R, the number of branched points, the number of exceptional points and the orbits of exceptional points from the structure of the KMS states for the gauge action. The talk is based on a joint work with M. Izumi and T. Kajiwara.