NEW METHODS FOR FAST SMALL-SIGNAL STABILITY ASSESSMENT OF LARGE SCALE POWER SYSTEMS

Leonardo T. G. Lima[1] Member
Licio H. Bezerra[2]
Carlos Tomei[3]
Nelson Martins[4] Senior Member

1 UNIVERSIDADE FEDERAL FLUMINENSE, Dept. of Electrical Engineering, R. Passo da Pátria 156, CEP 24.210, Niterói, RJ, BRAZIL
2 UNIVERSIDADE FEDERAL DE SANTA CATARINA, Dept. of Mathematics, CEP 88.040-900, Florianópolis, SC, BRAZIL, e-mail: mtm1lhb@ibm.ufsc.br
3 PONTIFÍCIA UNIVERSIDADE CATÓLICA, Dept. of Mathematics, Rio de Janeiro, RJ, BRAZIL, e-mail: tomei@mat.puc-rio.br
4 CEPEL, Caixa Postal 2754, CEP 20.001-970, Rio de Janeiro, RJ, BRAZIL, Fax. +55-21-260-1340, e-mail: pacdyn@acsi.cepel.br

Abstract *- This paper describes new matrix transformations suited to the efficient calculation of critical eigenvalues of large scale power system dynamic models. The key advantage of these methods is their ability to converge to the critical eigenvalues (unstable or low damped) of the system almost independently of the given initial estimate. Matrix transforms such as inverse iteration and S-matrix can be thought as special cases of the described method. These transforms can also be used to inhibit convergence to a known eigenvalue, yielding better overall efficiency when finding several eigenvalues.*

Keywords *- Small-signal stability, low damped oscillations, large scale systems, sparse eigenanalysis, matrix transforms.*

I. INTRODUCTION

Fast stability assessment is still a major concern for engineers engaged in large scale power systems operation. There is a need for the development of efficient real-time stability functions to be included in modern EMS.

Efficient methods for the small-signal stability analysis of large scale power systems were developed in the last decade [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Most of these methods are based in the use of the *augmented system equations* [2, 5, 6, 7, 8], exploiting the Jacobian matrix sparsity, and rely on iterative methods to obtain one or a few eigenvalues at a time [2, 4, 5, 10].

The major drawback of these methods is the difficulty to ensure that all unstable or low damped eigenvalues of the system have been found. One way to overcome this problem is the use of the **S**-matrix [3], a special matrix transform which maps the eigenvalues in the left half-plane to the circle of unitary radius. The unstable eigenvalues are therefore the eigenvalues of greater modulus and could then be calculated by a plain power method [11, 12] applied to **S**. Unfortunately the power method converges slowly due to the closeness of the moduli of the eigenvalues of **S** to one.

This paper describes new matrix transforms that overcome this problem, yielding a better convergence rate:

- inverse iteration applied to the **S**-matrix [13];
- power method applied to a Möbius [14] (i.e., linear fractional) transform of the **A** matrix;
- an efficient and highly effective method to inhibit convergence to already known eigenpairs [13]. This technique harnesses the convergence properties of partial eigensolution methods.

The Möbius transform allows the choice of three parameters, which can be used to modify the mapping properties. For instance, one can enhance convergence of eigenvalues within a certain region in the complex plane while simultaneously inhibiting convergence on another pre-specified region.

All methods rely on the flexibility in handling separately regions of spectrum by making use of iterations of conveniently chosen functions of the state matrix **A**.

The efficient implementation of these methods is obtained by expressing the basic step of each of these matrix transforms as the solution of a single linear system, with practically the same computational cost of an inverse iteration step [13].

The eigenvalue mapping properties of these matrix transforms are exemplified through a small test system. Results on the large scale Brazilian Interconnected System are then presented.

II. ITERATIVE EIGENVALUE COMPUTATION

Power Method

The basic idea that underlies almost every partial eigenvalue computation method is that the sequence $\mathbf{x}$, $\mathbf{Ax}$, ..., $\mathbf{A}^k\mathbf{x}$ converges to the eigenvector $\mathbf{q}_1$ associated with the eigenvalue of largest modulus (λ_1) of matrix **A**, provided that $|\lambda_1| > |\lambda_2| \geq ... \geq |\lambda_n|$ [11, 12]. The convergence of this method is linear and depends on the ratio

$$\frac{|\lambda_1|}{|\lambda_2|} \tag{1}$$

This method is not suitable for direct application to the small-signal stability analysis, since the modes of interest in this problem are not those with largest moduli in the state matrix **A**.

The inverse iteration method [11, 12] has been successfully applied to the small-signal stability analysis [2, 5]. This method uses the matrix transform

$$\mathbf{M}_1 = f_1(\mathbf{A}) = (\mathbf{A} - q\mathbf{I})^{-1} \tag{2}$$

where q is a complex shift, in place of the matrix **A**, in the power sequence. The eigenvalues of **A** closest to q will be mapped to the eigenvalues of largest moduli in $\mathbf{M}_1$ and thus the convergence will be driven to these eigenvalues and respective eigenvectors.

The **S**-matrix method proposed in [3] may be generalized to the matrix transform [13]

$$\mathbf{M}_2 = f_2(\mathbf{A}) = (\mathbf{A} + \bar{h}\mathbf{I})(\mathbf{A} - h\mathbf{I})^{-1} \tag{3}$$

where h is a complex number.

Although its initial application with the Lanczos method [3], this matrix transform could also be used with the power method to converge to the eigenvalue of largest modulus in $\mathbf{M}_2$.

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Spectral Transforms

Inverse iteration and **S**-matrix methods can be seen as power method applied to special functions of the state matrix **A** that suitably modify the spectrum of the original matrix **A**, allowing convergence to desired eigenvalues.

Any analytic function of the matrix **A** can be used with the power method (provided the evaluation of the power sequence is affordable). Therefore, matrix transforms suited to the eigenvalue problem in hand can be defined.

Möbius transform of the matrix **A** can be expressed as

$$\mathbf{M} = f_M(\mathbf{A}) = (a\mathbf{A} + b\mathbf{I})(c\mathbf{A} + d\mathbf{I})^{-1} = k_1(\mathbf{A} + k_2\mathbf{I})(\mathbf{A} + k_3\mathbf{I})^{-1} \quad (4)$$

where a, b, c and d are complex constants ($ad\text{-}bc \neq 0$) and **I** is the identity matrix. The complex parameters k_1, k_2 and k_3 are defined as

$$k_1 = \frac{a}{c} \quad , \quad k_2 = \frac{b}{a} \quad , \quad k_3 = \frac{d}{c} \quad (5)$$

This transform yields a power method iterative step with only one resolution of a set of linear algebraic equations in the form $\mathbf{Cx} = \mathbf{b}$, as in the inverse iteration and **S**-matrix methods.

It can be easily seen that the matrices **A** and **M** have the same eigenvectors and the eigenvalues of **M** are related to those of **A** by the same function that defines **M**:

$$\lambda_M = f_M(\lambda_A) = \frac{a\lambda_A + b}{c\lambda_A + d} = k_1 \frac{\lambda_A + k_2}{\lambda_A + k_3} \quad (6)$$

The power method can be readily applied to obtain the eigenvectors associated with the eigenvalues of largest moduli of the matrix transformation **M** [11, 13].

Each step of the power method requires a matrix-vector multiplication, which can be efficiently evaluated as follows:

$$\begin{aligned}\mathbf{Mx} &= (a\mathbf{A} + b\mathbf{I})(c\mathbf{A} + d\mathbf{I})^{-1}\mathbf{x} = \\ &= \frac{a}{c}\left\{\mathbf{I} + \left(\frac{b}{a} - \frac{d}{c}\right)\left(\mathbf{A} + \frac{d}{c}\mathbf{I}\right)^{-1}\right\}\mathbf{x} = \\ &= \frac{a}{c}\mathbf{x} + \left(\frac{bc - ad}{c^2}\right)\mathbf{w}\end{aligned} \quad (7)$$

where

$$\left(\mathbf{A} + \frac{d}{c}\mathbf{I}\right)\mathbf{w} = \mathbf{x} \quad (8)$$

Therefore, the basic power method applied to the Möbius transform could be described as follows:

Power Method with Möbius Transform Algorithm

1. Given the a, b, c, d parameters that define the Möbius transform, provide an initial estimate $\mathbf{x}_k$ for the eigenvector, where k is the iteration counter.
2. Solve $\left(\mathbf{A} + \frac{d}{c}\mathbf{I}\right)\mathbf{w}_k = \mathbf{x}_k$
3. Compute $\mathbf{z_k} = \frac{a}{c}\mathbf{x_k} + \frac{bc - ad}{c^2}\mathbf{w}_k$
4. Obtain new estimates for the eigenvalue/eigenvector pair
$$\mathbf{x}_{k+1} = \frac{\mathbf{z}_k}{\alpha_k} \quad , \quad q^M_{k+1} = \frac{1}{\alpha_k} \Rightarrow q^A_{k+1} = f_M^{-1}\left(q^M_{k+1}\right)$$
where α_k is the element of the vector $\mathbf{z}_k$ with maximum modulus.
5. Test the convergence of the method by a chosen criterion, e.g., calculate the residue
$$\mathbf{r}_{k+1} = \left(\mathbf{A} - q^A_{k+1}\mathbf{I}\right)\mathbf{x}_{k+1}$$
If $\|\mathbf{r}_{k+1}\|_\infty$ is greater than a tolerance, update iteration counter $k = k + 1$ and return to step 2.
Otherwise the algorithm has converged to the eigenvalue of greatest modulus in **M** and to its associated eigenvector.

Spectral Transform Definition

One must note that both inverse iteration and **S**-matrix methods are particular cases of a Möbius transform with predetermined parameters a, b, c, d. In both cases, there is only one parameter to be selected to achieve convergence to the desired eigenvalue.

More powerful matrix transformations can be obtained by conveniently choosing the parameters a, b, c, d. As a matter of fact, there are three degrees of freedom in the definition of a Möbius transform, though one can pick three points in the complex plane (μ_1^A, μ_2^A and μ_3^A) and assign their transformed values (μ_1^M, μ_2^M and μ_3^M). Thus, the Möbius transform is defined by solving the simultaneous equations [14]

$$\mu_i^{\mathbf{M}} = k_1 \frac{\mu_i^{\mathbf{A}} + k_2}{\mu_i^{\mathbf{A}} + k_3} \quad , \quad i = 1 \ldots 3 \quad (9)$$

for the unknown parameters k_1, k_2 and k_3.

Note that the choice of the points μ_i^A depends on the region in the complex plane where the desired eigenvalues are located.

One special Möbius transform definition is the following:

- send a point μ_1 to ∞ by setting $k_3 = -\mu_1$ (equivalent to enhance convergence to eigenvalues in its neighborhood, as in the inverse iteration method);
- send a point μ_2 to 0 by setting $k_2 = -\mu_2$ (equivalent to inhibit convergence to eigenvalues in this neighborhood).

Furthermore, all the points in the mediatrix of the segment defined by the points μ_1 and μ_2 will be mapped to a value with constant modulus equal to $|k_1|$.

Another special Möbius transform that can be easily defined is that involved in the inverse iteration with matrix **S**:

$$\begin{aligned}\mathbf{M}_3 &= f_3(\mathbf{A}) = (\mathbf{S} - q\mathbf{I})^{-1} = \left[(\mathbf{A} + \bar{h}\mathbf{I})(\mathbf{A} - h\mathbf{I})^{-1} - q\mathbf{I}\right]^{-1} = \\ &= \frac{1}{1-q}\left[\mathbf{I} + \frac{2\mathrm{Re}(h)}{1-q}\left(\mathbf{A} + \frac{\bar{h} + qh}{1-q}\mathbf{I}\right)^{-1}\right]\end{aligned} \quad (10)$$

where h and q are complex constants.

It can be readily seen that this is a Möbius transform expressed in the same form of equation (7).

Table 1 presents the choices of parameters to define the various matrix transforms presented in this paper through use of a Möbius transform.

Matrix Transform	Möbius Transform Parameters			
	a	b	c	d
Inverse Iteration on **A** ($\mathbf{M}_1$)	0	1	1	$-q$
S-matrix ($\mathbf{M}_2$)	1	$\bar{h}$	1	$-h$
Inverse Iteration on **S** ($\mathbf{M}_3$)	1	$\frac{2\bar{h} + h + qh}{1-q}$	$1-q$	$\bar{h} + qh$

Table 1. Special Möbius Transforms Parameters Definition

III. INHIBITING CONVERGENCE FOR EIGENVALUES

A vector **x** may be expressed as a linear combination of the eigenvectors of matrix **A** (provided that **A** is non-defective):

$$\mathbf{x} = \sum_i c_i \mathbf{q}_i \quad (11)$$

where $\mathbf{q}_1, ..., \mathbf{q}_n$ are eigenvectors associated with $|\lambda_1| > |\lambda_2| > ... > |\lambda_n|$.

The power method applied to the vector $\mathbf{x}$ will converge to the eigenpair $(\lambda_1, \mathbf{q}_1)$, if $c_1 \neq 0$.

Once the first eigenpair is obtained, the power method may be restarted to search for λ_2 with the iteration vector occasionally multiplied by $(\mathbf{A} - \lambda_1\mathbf{I})$. This procedure will inhibit the eigenvector direction $\mathbf{q}_1$ due to the product:

$$(\mathbf{A}-\lambda_1\mathbf{I})^k\mathbf{x} = \sum_{i=1}^{n} c_i(\lambda_i-\lambda_1)^k\mathbf{q}_i = \sum_{i=2}^{n} c_i(\lambda_i-\lambda_1)^k\mathbf{q}_i \quad (12)$$

This procedure can be readily applied in association with the matrix transforms described in Section II yielding a new transform of the state matrix $\mathbf{A}$.

The new matrix transform

$$f_\lambda(\mathbf{A}) = \mathbf{M}(\mathbf{A}-\lambda\mathbf{I}) \quad (13)$$

can thus be used to inhibit convergence for a known eigenvalue λ during power method iteration without changing the pre-determined Möbius transform $\mathbf{M}$.

IV. NEW ENGLAND TEST SYSTEM RESULTS

The New England Test System has been widely used as a benchmark model in power system stability analysis [1, 2, 4, 9, 15]. The state matrix has 66 eigenvalues, whose numeric values are the same as those of [2, 9] and differ slightly from those of [1, 4] since speed-governor and exciter saturation effects were here neglected.

Table 2 lists the nine electromechanical modes of interest of the New England Test System (least damped eigenvalues) and the generators with highest participation in each mode.

	Eigenvalue	Generators
1	$-0.467 \pm j\,8.965$	36, 35
2	$-0.412 \pm j\,8.779$	37
3	$-0.370 \pm j\,8.611$	33
4	$-0.282 \pm j\,7.537$	32, 31
5	$-0.112 \pm j\,7.095$	30
6	$-0.297 \pm j\,6.956$	35, 36, 31
7	$-0.283 \pm j\,6.282$	31, 32, 34, 38
8	$-0.301 \pm j\,5.792$	38, 34
9	$-0.249 \pm j\,3.686$	39, 38, 34

Table 2. New England Test System Critical Eigenvalues

This Section will present some surface plots to highlight the effect of different Möbius transforms over the spectrum of the state matrix $\mathbf{A}$ of New England Test System. Each plot represents the modulus of the transform evaluated over a mesh defined in a small region of the complex plane. A contour plot including the eigenvalues of the system is also presented together with its surface plot.

Figure 1 shows the plots for the inverse iteration transform ($\mathbf{M}_1$) with $q = j\,7$. Only three eigenvalues close to the given shift are transformed to moduli greater than 1.5 and thus the convergence will occur for one of them.

Figure 2 presents the plots for the S-matrix transform ($\mathbf{M}_2$) with $h = 7 + j\,5$. Note that almost all the eigenvalues have their moduli in the range 0.8 to 1. This is a characteristic of this transform and will slow down the convergence of the power method, as can be seen from equation (1). It is also worth of mention the fact that all points in the imaginary axis have modulus equal 1, i.e., that axis was mapped as the circle of unitary radius, as expected [3].

The plots for the inverse iteration on matrix $\mathbf{S}$ ($\mathbf{M}_3$) with $h = 7 + j\,7$ and $q = -1$ are shown in Figure 3. Note that these figures are quite similar to those for the inverse iteration on matrix $\mathbf{A}$ (figure 1), but for the scale. This is due to a higher gradient of the modulus of the $\mathbf{M}_3$ function near the singularity. This characteristic may yields a better convergence of the power method with this transform.

The plots for a special Möbius transform are presented in Figure 4. The zero of the transform ($-k_2$) was calculated as a function of the given pole ($-k_3$) to assure the mapping of the line of constant damp $\xi = 0.1$ to a constant modulus equal 1, i.e., all eigenvalues with a damp greater than 0.1 will be mapped to a modulus less than 1. Therefore, the convergence of the power method with this transform will occur to low damped or unstable eigenvalues only.

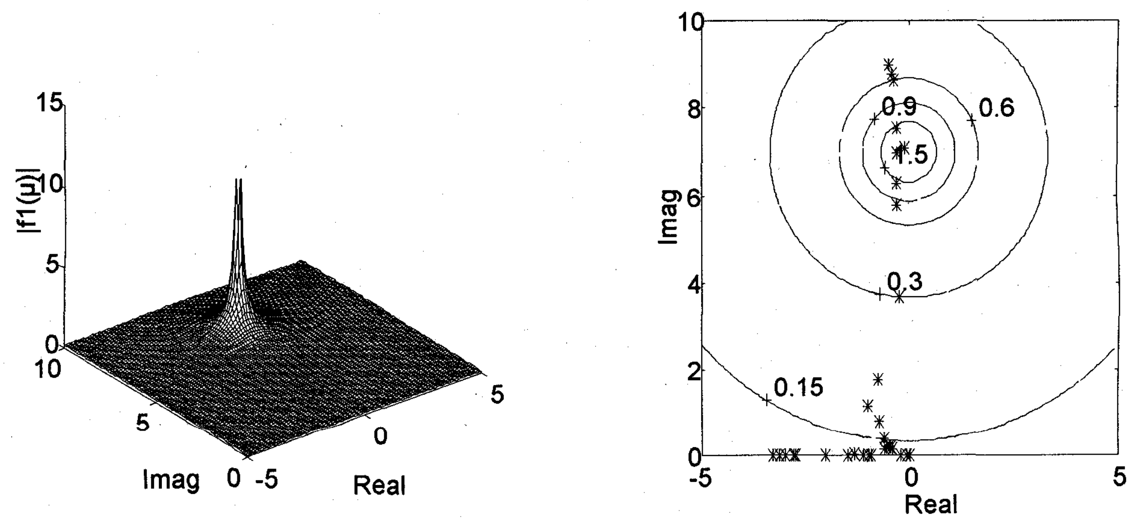

Figure 1. Mapping Properties of Inverse Iteration on Matrix $\mathbf{A}$ (Matrix Transform $\mathbf{M}_1$) ($q = j\,7$)

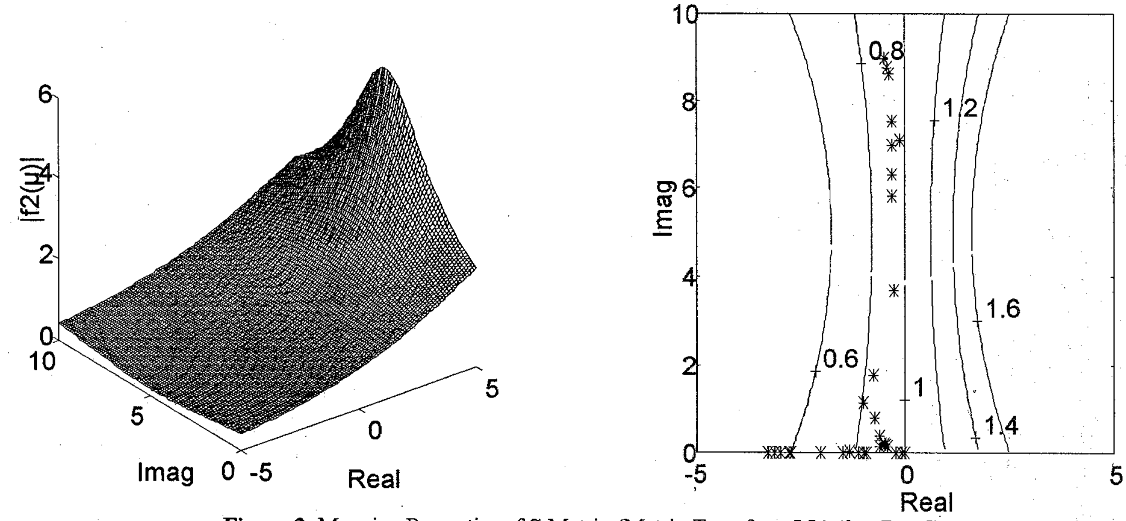

Figure 2. Mapping Properties of **S**-Matrix (Matrix Transform $\mathbf{M}_2$) ($h = 7 + j\,5$)

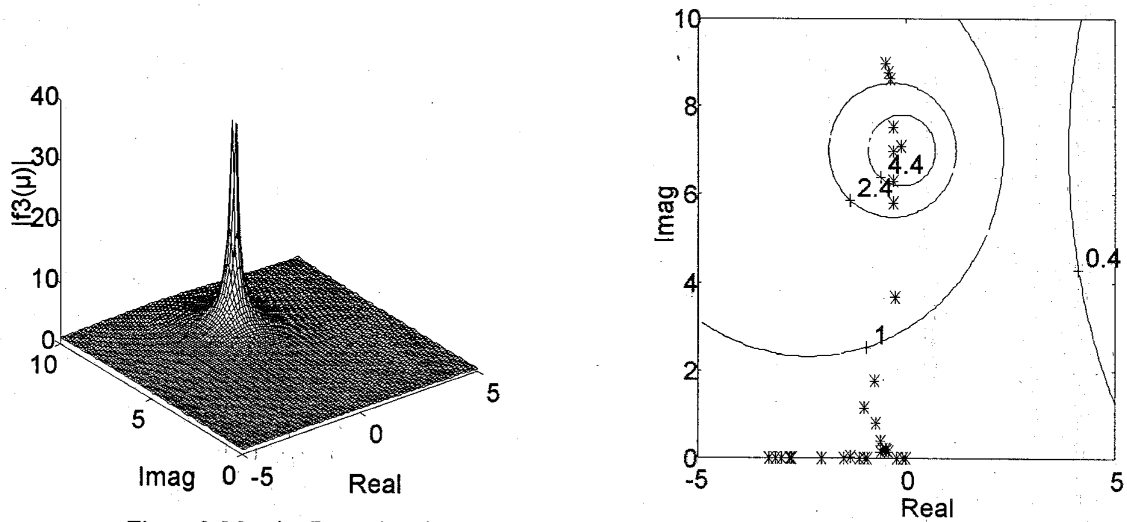

Figure 3. Mapping Properties of Inverse Iteration on **S** (Matrix Transform $\mathbf{M}_3$) ($h = 7 + j\,7$; $q = -1$)

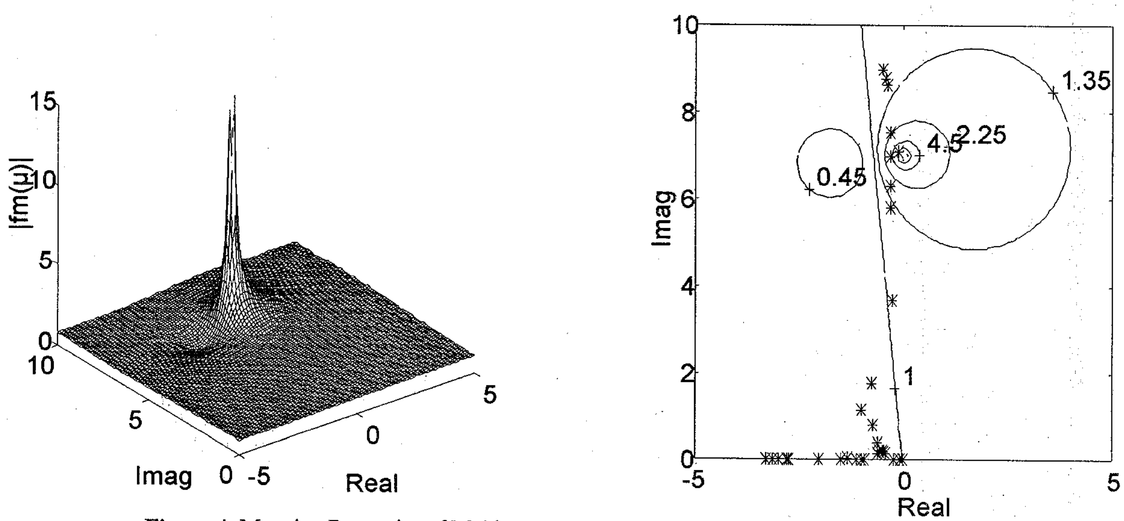

Figure 4. Mapping Properties of Möbius Transform ($a = 1$; $b = 1.39298 - j\,6.86$; $c = 1$; $d = -j\,7$)

V. LARGE SCALE SYSTEM RESULTS

The large scale test system used is derived from a practical stability model for the Brazilian South-Southeast Interconnected System having 1477 buses, 2200 lines, 103 generators, 1 HVDC link, and 272 induction motors. The Jacobian matrix for this system model has 7037 equations with 1976 state variables.

To search for critical eigenvalues, points over the imaginary axis were used as poles for the Möbius transform (shifts for inverse iteration on **A**). These poles varied from 1 rad/s to 10 rad/s with a step of 1 rad/s.

The following matrix transforms were applied:

- inverse iteration on matrix **A** ($\mathbf{M}_1$);
- inverse iteration on matrix **S** ($\mathbf{M}_3$);
- Möbius transform with a zero in the imaginary axis of 1 rad/s less than the pole. All eigenvalues with imaginary part less than the pole frequency minus 0.5 rad/s have modulus less than 1.
- Möbius transform with a zero such that the line of constant damp $\xi = 0.1$ is mapped to the circle of unitary radius, i.e., all eigenvalues with $\xi > 0.1$ will have modulus less than 1.

These matrix transforms were applied with a Simultaneous Iteration algorithm [5, 10, 11] limited to 10 vectors, 10 iterations, each iteration composed of a fast iteration cycle of 3 power method steps and a full complex matrix (10x10) eigensolution.

Table 3 presents the results obtained. A convergence tolerance of 10^{-6} for the ∞-norm of the residue vector was utilized for all the shown eigenvalues. This is a very tight convergence criterion that could be relaxed for most practical studies.

The Möbius transforms converged less eigenvalues than inverse iteration on **A** and on **S**. This was expected due to the numerator in the Möbius transform that inhibits convergence on a certain region of the complex plane. On the other hand, these transforms behaved exactly as expected, i.e., the eigenvalues obtained by them are located in the half plane defined by the mediatrix of the segment defined by the pole and the zero of the transform:

- for the first Möbius transform, only eigenvalues with imaginary part greater than (pole + zero)/2 were detected;
- for the second one, only eigenvalues with $\xi < 0.1$ were detected.

complex shift	INVERSE ITERATION ON A			INVERSE ITERATION ON S			MÖBIUS (1)			MÖBIUS (2)		
	Converged Eigenvalue		iter	Converged Eigenvalue		iter	Converged Eigenvalue		iter	Converged Eigenvalue		iter
	real	imag	#	real	imag	#	real	imag	#	real	imag	#
1	-0.3542681	0.9524427	8	-0.3542681	-0.9524427	8	-0.3542681	0.9524427	10			
2	-0.492743	1.793309	4	-0.492743	-1.793309	4						
	-1.027465	1.976434	10	-1.027463	-1.976431	8						
3	-0.2831537	3.462319	4	-0.2831537	-3.462319	4	-0.2831537	3.462319	7			
	-1.027491	1.976458	9	-1.165506	-3.777869	8						
	-1.165512	3.777867	10	-0.5074013	-4.526962	9						
	-0.4927435	1.793309	10									
4	-0.2831537	3.462319	4	-0.2831537	-3.462319	5	-0.5073678	4.526945	9			
	-1.165507	3.777869	7	-1.165507	-3.777867	9	-7.59E-02	4.730554	8			
	-0.5073684	4.526944	5	-0.5073684	-4.526944	5						
	-7.59E-02	4.730554	4	-7.59E-02	-4.730554	5						
	-0.169365'	5.104331	7	-0.1473733	-5.097544	8						
	-0.1473725	5.097544	7	-0.1693624	-5.104332	8						
5	-0.1473728	5.097544	3	-0.1473728	-5.097544	3	-0.1473728	5.097544	4	-0.1473728	5.097544	5
	-7.59E-02	4.730554	3	-0.1693647	-5.104332	3	-7.59E-02	4.730554	8	-0.1693647	5.104332	5
	-0.1693647	5.104332	3	-7.59E-02	-4.730554	3	-0.1693647	5.104332	4	-7.59E-02	4.730554	5
	-0.5073684	4.526944	5	-0.5073684	-4.526944	5	-0.1784199	5.629812	9			
	-0.4093778	5.743621	6	-0.4093778	-5.743621	7						
	-0.17842	5.629811	5	-0.17842	-5.629811	6						
6	-0.17842	5.629811	5	-0.17842	-5.629811	6	-0.1069953	6.373759	7	-0.17842	5.629811	8
	-0.4093778	5.743621	5	-0.4093778	-5.743621	6	-0.2460411	6.368391	7	-0.2460411	6.368391	8
	-0.2280641	6.773929	9	-0.2280112	-6.773918	9				-0.1069953	6.373759	7
	-0.2460411	6.368391	5	-0.2460411	-6.368391	6						
	-0.1069953	6.373759	5	-0.1069953	-6.373759	5						
7	-0.1661218	7.114723	3	-0.1661218	-7.114723	3	-0.1661218	7.114723	4	-0.1661218	7.114723	4
	-0.3504729	7.341525	8	-0.3504761	-7.34152	9	-0.1949145	6.815865	7	-0.1069955	6.373759	10
	-0.4864469	7.150498	8	-0.4864479	-7.150497	8	-0.2280633	6.773909	10	-0.3504738	7.341526	9
	-0.4097608	7.068548	6	-0.4097608	-7.068548	6	-0.3504735	7.341525	9	-0.4097608	7.068548	9
	-0.2280635	6.773909	4	-0.2280635	-6.773909	4	-0.409761	7.068548	9	-0.1949145	6.815865	5
	-0.1949145	6.815865	4	-0.1949145	-6.815865	4				-0.2280635	6.773909	5
8	-0.3503849	8.334722	7	-0.40971	-7.75503	5	-0.3503848	8.334723	9	-0.4097101	7.75503	8
	-0.40971	7.75503	5	-0.5059292	-8.287663	9				-0.3503849	8.334722	8
	-0.5059293	8.287663	9	-0.3503849	-8.334722	6						
9	-0.249553	8.681234	6	-0.249553	-8.681234	6	-0.2727971	9.681756	7	-0.249553	8.681234	6
	-0.4741303	8.602891	8	-0.4741309	-8.602891	8				-0.4741304	8.60289	9
	-0.613992	8.860255	10	-0.2727946	-9.681758	9				-0.2727972	9.681755	7
10	-0.1042566	10.44917	4	-0.1042566	-10.44917	4	-0.1042566	10.44917	6	-0.2727972	9.681755	5
	-0.2727972	9.681755	3	-0.2727972	-9.681755	3				-0.1042566	10.44917	5
	-0.7103217	9.636901	10	-0.7103293	-9.636911	10						

Table 3. Brazilian Interconnected System Eigenvalues Calculated by Different Matrix Transforms

VI. FINAL COMMENTS

This paper dealt with new algorithms developed for small-signal stability analysis by eigenvalue methods. New matrix transforms - Möbius transforms - were defined so that inverse iteration and **S**-matrix methods were particular cases of power method applied to these transforms. It was shown that power method with these Möbius transforms have the same computational cost as inverse iteration with the state matrix **A**.

The Möbius transforms are the result of the extension to matrices of the map $f(x)=\dfrac{ax+b}{cx+d}$, $ad - bc \neq 0$. Therefore, power method applied to $f(\mathbf{A})$ will converge to the eigenvalues closest to ($k_3 = -d/c$) while inhibiting convergence to eigenvalues close to ($k_2 = -b/a$) if $a \neq 0$. With suitable definition of the parameters a, b, c, d, an improved control over eigenvalue calculation may be achieved.

The numerical tests indicated that those transforms were efficient when convergence to low damped eigenvalues were desirable. Tests conducted with a large power system model clearly demonstrated how the use of different transforms could focuse the convergence of the power method.

The inverse iteration method (power method applied to a particular Möbius transform) allows convergence only to eigenvalues in a region close to a point in the complex plane. A Möbius transform extends this characteristic to allow a second degree of freedom: the definition of another point such that convergence is inhibited in its neighborhood.

The control of convergence of partial eigensolution methods through use of more general Möbius transforms indicates that this is a new powerful tool to be applied to small-signal stability analysis and equivalent sparse eigenproblems.

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Leonardo T. G. Lima (M'93) received his B.Sc. (1986) and M.Sc. (1991), both in electrical engineering, from COPPE/UFRJ, Brazil. Mr. Lima is a lecturer at the Dept. of Electrical Engineering, Fluminense Federal University, and is presently working towards a D.Sc. degree at COPPE/UFRJ. His current work and interests include power system dynamics, control theory applications and the development of large scale power system analysis tools.

Licio H. Bezerra received his B.Sc. (1974), M.Sc. (1979) and D.Sc. (1990), in mathematics, from Pontificia Universidade Católica do Rio de Janeiro, Brazil. Mr. Bezerra is an assistant professor at the Dept. of Mathematics of Federal University of Santa Catarina. His current work and interests include numerical linear algebra, optimization and numerical resolution of partial differential equations.

Carlos Tomei received his B.Sc. (1975) and M.Sc. (1978), both in mathematics, from Pontificia Universidade Católica do Rio de Janeiro, Brazil, and Ph.D. (1982) from the New York University, Courant Institute of Mathematics, USA. Mr. Tomei is an Associate Professor of Pontificia Universidade Católica do Rio de Janeiro and his main research interests are inverse scattering theory and numerical analysis.

Nelson Martins (M'81, SM'91) received the B.Sc. (1972) from the University of Brasilia, Brazil, the M.Sc. (1974) and Ph.D. (1978) degrees, from the University of Manchester, Institute of Science and Technology (UMIST), UK, all in electrical engineering. Mr. Martins works at CEPEL, the Brazilian electrical energy research center, in the development of methods and computer tools for power system dynamics and control.

Discussion

G. Angelidis and **A. Semlyen** (University of Toronto): We wish to commend the authors for their interesting paper. Although the concepts presented are not new, the paper has practical and tutorial value. We note that the S-matrix has been known for a long time as the Cayley transform of a matrix. Since the transformations are used in the paper in conjunction with the power method, where the modulus of the eigenvalue is of interest, the graphical approach used by the authors of representing a transform by surface and contour plots is very instructive. In practical applications, it seems however that some Lanczos or Arnoldi-type method should be used instead of the power method.

The spectral transformation of the Möbius transform can be more easily visualized if the three points μ_1, μ_2 and μ_3, are viewed as defining a circle (or a straight line). Thus the transform maps circles to circles. The objective is to have concentric circles in the **M**-plane (the spectral domain of **M**). In the case of the Cayley transform, these result from the mapping of Apollonius circles in the **A**-plane. In the limit, as suggested in the paper, the mediatrix (locus of equidistant points) to the foci of the Apollonius circles in the **A**-plane is mapped to the unit-circle in the **M**-plane.

The deflation by pre-multiplication of the working vector **x** by the matrix $(\mathbf{A}-\lambda_1\mathbf{I})$ is a very good and simple idea. This matrix-vector multiplication eliminates the component of **x** in the direction of $\mathbf{q}_1$, as indicated by (12). We have used successfully an alternative technique for implicit deflation in our sequential eigenanalysis algorithms [A]. According to this technique, Schur vectors, instead of eigenvectors, are calculated. The working vector is orthogonalized to the set of previously calculated Schur vectors. The orthogonalization is computationally much less demanding than a sparse matrix-vector product. The process for retrieving the eigenvectors from the Schur vectors is trivial.

The authors' comments would be greatly appreciated.

[A] G. Angelidis and A. Semlyen, "Efficient Calculation of Critical Eigenvalue Clusters in the Small Signal Stability Analysis of Large Power Systems", Paper No. 94 SM 556-1 PWRS, presented at the IEEE/PES Summer Meeting, San Francisco, California, July 1994.

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Leonardo T. G. Lima, Licio H. Bezerra, Carlos Tomei, Nelson Martins (UFF - Niterói, RJ; UFSC - Florianópolis, SC; PUC-RJ - Rio de Janeiro, RJ; CEPEL - Rio de Janeiro, RJ, Brazil) - We thank Prof. Semlyen and Dr. Angelidis for their valuable comments and questions. We will try to address each point in the same order as they were raised.

Cayley and Möbius transforms are well known and have been applied in other engineering fields. The proposal of this paper which is believed to be original is the use of these transforms with varying parameters and even mix different transforms in a single partial eigensolution run to achieve better convergence characteristics and/or to yield improved control over the eigenvalues (of **A**) of interest.

The power method was used to simplify the description of each transform. The large scale system results in section 5 were all obtained through lop-sided simultaneous iteration [10, 11, 13].

The choice of parameters *a, b, c* and *d* is the key point to the successful application of the Möbius transform to the iterative eigenvalue finding problem.

The selection of the parameters requires the simultaneous solution of three nonlinear equations, shown in equation (9). This set of equations will have a solution only if three different points in the **A**-plane are specified to be mapped to three different points in the **M**-plane (including 0 and ∞). Obviously, the same parameters selection can be achieved from different sets of specified points.

One of the best applications of the Möbius transform is the implicit deflation by pre-multiplying the working vector **x** by the matrix $(\mathbf{A}-\lambda_i\mathbf{I})$. Suppose one is using the inverse iteration method, defined by the matrix transform $(\mathbf{A}-q\mathbf{I})^{-1}$. The inverse iteration with the implicit deflation iterative step may be defined as

$$\begin{aligned}\mathbf{w} &= (\mathbf{A}-\mathrm{q}\mathbf{I})^{-1}(\mathbf{A}-\lambda_i\mathbf{I})\mathbf{x} = \\ &= (\mathbf{A}-\mathrm{q}\mathbf{I})^{-1}(\mathbf{A}-\lambda_i\mathbf{I}+\mathrm{q}\mathbf{I}-\mathrm{q}\mathbf{I})\mathbf{x} = \\ &= \left[\mathbf{I}+(\mathrm{q}-\lambda_i)(\mathbf{A}-\mathrm{q}\mathbf{I})^{-1}\right]\mathbf{x} = \\ &= \mathbf{x}+(\mathrm{q}-\lambda_i)(\mathbf{A}-\mathrm{q}\mathbf{I})^{-1}\,\mathbf{x}\end{aligned}$$

which is a Möbius transform as posed in (7). This derivation was ommited in the text of the paper for brevity. However, as one of the reviewers found it relevant we take the opportunity to include it in this closure.

The computational effort demanded here is equivalent to that required for the inverse iteration method, with an extra sum of two vectors. Note that the matrix-vector multiplication $(\mathbf{A}-q\mathbf{I})^{-1}\mathbf{x}$ is carried out through the solution of the equation $(\mathbf{A}-q\mathbf{I})\mathbf{w}=\mathbf{x}$ and uses the **LU** decomposition to fully exploit the sparse nature of the problem. There is no need to refactorize the matrix to apply the implicit deflation. This procedure demands less computational effort than other deflation methods based on orthogonalization algorithms. The robustness and convergence characteristics of the method are to be evaluated in order to establish its overall performance when compared to other deflation algorithms.

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